Vera Effiges Venteri M

R. White Sculp.

Vera Effigies Venteri Mandey.
Etatis Suæ 37.

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Vera Effigies Venteri Mandey.
Etatis Suæ 37.

MELLIFICIUM MENSIONIS:
OR, THE
MARROW of MEASURING.

WHEREIN

A new and ready way is shewn how to measure *Glazing*, *Painting*, *Plastering*, *Masonry*, *Joyners*, *Carpenters*, and *Bricklayers* Works.

AS ALSO

The *Measuring* of *LAND*, and all other *Superficies* and *Solids*, by *Vulgar Arithmetick*, without reducing the *Integers* into the least Denomination; giving the Content of any *Superficies* or *Solid*, consisting of *Feet*, *Inches*, and *Parts of Inches*, in a fourth Part of the *Time* and *Labour* required by the usual way in *Vulgar Arithmetic*.

TOGETHER

With some CHOICE PRINCIPLES and PROBLEMS of *GEOMETRY* conducing thereto.

The whole Treatise *being comprized in six Books, and illustrated with Copper Cuts.*

An account whereof is given in the Epistle to the Reader; *the like not heretofore published.*

By VENTERUS MANDEY.

Non Quot, sed Quales.

London, Printed for the Author, and sold by *Richard Chiswel*, at the *Rose* and *Crown* in St. *Paul's* Church-yard; and *Benjamin Clark*, in *George-Yard* in *Lumbard-Street*; and by the Author, at the *Red-Heart* in *Heart-Street* in *Bloomsbury Market*; 1682.

TO THE HONOURABLE

Sir JAMES BUTLER,

KNIGHT,

Steward of the Court of the *Marſhalſea* of the *King's Houſhold*, one of the Judges of His Majeſty's *Palace Court*, one of the King's Council at *Law*, and Attorney General to the Queen's Majeſty, *&c.*

Honoured Sir,

THE ſeveral Offices of Honour and Truſt conferred upon You by our moſt Gracious Soveraign, and Your Loyalty, Integrity, and Courage, in diſcharging of the ſame, do abundantly manifeſt Your Merits: And for me to declare or ſet forth Your Vertues, were but to tell the World, that it is day-light when the Sun is upon the Meridian.

The Subject of the enſuing Treatiſe being Mathematical, *I humbly preſume to implore Your Honours Patronage of it, as being a Science chiefly invented for doing of Juſtice, and perfor-*

mance of that which is Right between Man and Man, whereof Your Honour is a most competent Judge.

For by this Science of Mensuration, the Surveyor or Land-measurer limits and sets out the true Quantity of Land that one person buys of another: Also by this, the Quantity, and from thence the Value of several Artificers Works are justly Computed, and rightly Estimated.

The Excellency also of Geometry *(whereof partly the subsequent Treatise consists) appears, for as much that by it* Astrologians *make their Observations, by it they declare the extent of the Heavens, the motion of the Sun, Moon, Stars,* &c. *In brief,* Geography, Architecture, *and all other* Mathematical Sciences, *have a dependance upon it.*

Now if these Lucubrations of mine obtain but Your Honours Approbation, I shall think my self happy, and hope that with Your Name *and* Memory *this Work may remain to Eternity.*

May You daily increase in Honour and Glory, be repleniſhed with all Earthly Blessings, and for ever enjoy the full fruition of all Happiness, both in this World, and that which is to come, is the Prayer of

Your Honours most humble
and affectionate Servant,

VENTERUS MANDEY.

THE Author's Epiſtle TO THE READER.

Courteous Reader,

THe Subject of the enſuing Treatiſe is the Science of Meaſuring in a new and brief method by vulgar Arithmetic, wherein the laborious and troubleſom part of reducing the Feet and Inches into the leaſt denomination, is omitted, and a brief and ſpeedy way directed for the caſting up any Dimenſion whatſoever.

Which way of Meaſuring, conſidering that Workmens Rulers are not divided Decimally, and that moſt Dimenſions are taken by a Ruler divided into Feet, Inches, half Inches, and Quarters, and for the moſt part, the Contents are required in the ſame kind; I ſay, conſi-

dering this, it cometh very little ſhort of the Decimal way of Meaſuring.

For if Dimenſions be given in Feet, Inches, and halves or quarters of Inches, (as moſt uſually they are) and the Contents required in the ſame method, then I look upon the reducing thoſe Dimenſions into Decimals, and after the Contents are found in Decimals, the reducing them again into Feet, Inches, and half Inches, *&c.* to take up as much time as this way which is taught in the enſuing Treatiſe.

Beſides, many Men can multiply and divide by vulgar Arithmetic, which do not underſtand Decimals, for whoſe ſakes chiefly it is written.

I conceive every Book meets with many critical Cenſures, and I doubt not but this will partake with the reſt; and therefore it might (perchance) be expected, that I ſhould excuſe my ſelf for whatſoever any Man ſhall be pleaſed to object againſt, in it, which I ſhall neglect, only deſiring the Judicious Reader to paſs by ſome ſmall overſights which perhaps there may be crept into the enſuing Work, as knowing that all Men's doings are ſubject to error: But as for groſs faults, I think there are none, for I have been as careful as I could, both in writing, and likewiſe in correcting

the

the printed ſheets, conſidering my want of time occaſioned by my other employment.

To perſuade any one to the ſtudy of this Science would be a folly, ſince the Ignorant (which are blind) cannot judge truly; and to him that already underſtands it, the Labour would be uſeleſs and unprofitable; and to the averſe and careleſs, it would be like the caſting of Pearls before Swine; the exquiſite knowledge whereof cannot be attained by ſuch prejudicated perſons, although to many that are Judicious, the uſefulneſs hereof is already ſufficiently known; and to thoſe Induſtrious perſons which are yet to ſeek in the knowledge of this Subject, and deſire to be informed, this Treatiſe will fully anſwer their expectations.

In the following Treatiſe I have endeavoured to proceed methodically, and have to my knowledge omitted nothing which might tend to the making of a Man an expert Meaſurer; in order to which, there are three Books of *Geometry* inſerted, The firſt containing the Rudiments thereof, and the ſecond and third containing choice Problems, which two laſt I Tranſlated from *Latin* Copies, wherein I have endeavoured to render the meaning of the Author as plain as the Work would permit, and have amended ſome mi-

ſtakes (I ſuppoſe) of the printer of the *Latin* Copies; I have likewiſe explained the meaning of ſome difficult terms, and alſo added a new Diagram belonging to *Chap. 6. Lib. 3. Fig. 1.* which was wanting in the Copy.

So that the whole Treatiſe conſiſts of ſix Books, a general account whereof follows.

The firſt Book of this Treatiſe contains the Rudiments of *Geometry*, conſiſting of ſuch Definitions and Propoſitions as are meet to be known to any Man that intends the Science of this Subject, and is an Introduction to the other five Books; in which firſt Book (perhaps) ſome few of the Definitions may ſeem ſtrange, as not agreeing with the Definitions of ſome others, yet, in my opinion, agreable both to reaſon and truth: There is likewiſe added, the deſcription of Ovals to any length and breadth, with a pair of Compaſſes; and alſo a digreſſion concerning Ovaller Arches.

The ſecond Book, being tranſlated from a *Latin* Author, contains a Garden of *Geometrical Roſes*, conſiſting of choice Propoſitions in *Geometry*, wherein a new way is ſhewn of cutting right Lines in extream and mean Proportion; alſo the diviſion of Angles, and finding a ſtreight Line in length equal to a Circular; as alſo to find the Center of Gravity of a Semicircle or Quadrant, with ſeveral

ral other things not heretofore publiſhed in *Engliſh* to my knowledge.

The third Book, being likewiſe tranſlated from the ſame Author, contains ſome Principles and Problems in *Geometry*, formerly thought deſperate, now briefly explained in *Engliſh*, conſiſting of the Subject, Principles, and Method of the *Mathematics*, and of *Algebraical* Operations; likewiſe of a new method of treating of Solids and their Superficies, by the efficient cauſes; with ſeveral other things.

The fourth Book contains the Science of *Menſuration*, ſhewing how to meaſure all kind of works relating to Building, to wit, *Carpenters*, *Glaſiers*, *Painters*, *Plaſterers*, *Maſons*, and *Bricklayers* works, *&c.* the Contents whereof may be ſeen more at large in the following Table.

The fifth Book contains the meaſuring of *Superficial Plains*, wherein is ſhewn how to meaſure *Triangles* of all kinds, *Rectangled Figures* whoſe Sides are equal or unequal, *Circles*, *Ovals*, *Pyramids*, *Cones*, *&c.* In brief, it ſhews how to find the Content of all kind of Superficial Figures, whether they be regular or irregular, each Propoſition having a Figure or Diagram belonging to it, for the more eaſier underſtanding how to reſolve it.

This

This fifth Book alſo ſhews how to meaſure Land lying in any form, and how to reduce cuſtomary Meaſure into Statute Meaſure, and the contrary, *&c.*

The ſixth Book contains the meaſuring of *Solid Bodies*, ſhewing how to meaſure *Timber* and *Stone*, *&c.* alſo how to find the Solid Contents of *Pyramids*, *Cones*, *Cylinders*, *Spheres*, and other ſuch-like Solids, each Propoſition having a Diagram (or Scheme) belonging to it, for the readier attaining the Reſolution of the Propoſition. In it is alſo contained the Science of *Gauging*, or meaſuring *Liquid Veſſels*, and how to find their Contents in Wine or Ale Gallons; and likewiſe how to find the Contents of *Brewers* Veſſels, Tuns, or Fats, in Gallons, and to reduce them into Barrels; all which I have drawn into a Pocket Volume.

This Treatiſe thus finiſhed I preſent thee with, deſiring thy friendly Cenſure and Acceptance of theſe firſt Fruits of my Labours; as alſo to paſs by ſuch Faults as may poſſibly have eſcaped the Preſs, or my ſelf: And in ſo doing, thou wilt oblige him, who is

A Friend to all that are

Mathematically enclin'd,

V. M.

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The truth of the resolution to the foregoing Proposition demonstrated by the comparison of Motion. Page 364

Of the proportions between a Cube, a Prisma, and a Pyramid, a Cylinder, Sphere, and Cone, whose Heights and Bases are alike. 365

Gauging or measuring of Liquid Vessels, what it is, and how to measure any Vessel, and give the Content in Wine or Ale measure, &c. 367

UMI

Of GEOMETRY in general.

GEOMETRY is a Greek word, and imports only the meaſuring of Land; and ſince ſeveral parcels of Lands are divided into various forms, it behoves him that would be eſteemed a Land-meaſurer, to know how to meaſure all kinds of ſuperficial Figures. But though the word import no more but Land (or ſuperficial) meaſuring, yet under the name of GEOMETRY is compriſed the meaſuring of all kinds of ſolids. This Science (according to Hiſtory) was firſt invented by the *Egyptians*, and the cauſe which put them upon inventing it, was the over-flowing of the River *Nylus*, the greateſt and longeſt River in the World, which when it over-flowed, waſhed away their Banks and Land-marks, and when the Waters were diſperſed, it was

a difficult matter for every man to know his own quantity of Land, insomuch that it caused quarrelling and strife amongst them, till at length through the Industry of some ingenious Person or Persons, this Science of GEOMETRY was found out, which put an end to their quarrellings, and restored to every man the same quantity of Land after the Flood, that he had before.

In brief, GEOMETRY is a Science, whereby the quantities of things not measured are determined, by comparing them with other quantities measured.

OF

OF GEOMETRY.

CHAP. I.

Definitions.

I. A Point is a Body whose Quantity is not considered; if considered, is that which is not put to account in Demonstration; and is made with the point of the Compass, of a Pen or Pencil or such like, as the point noted by A.

II. A Line is a Body whose Length is considered without its Breadth, and is made by the motion of a point from one extream to another.

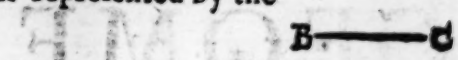
Extream, signifies the beginning or end of a Line.

Of Lines are Three Sorts.

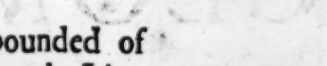
First, Crooked or Circular. Secondly, Streight or Right. Thirdly, Mixt.

1. Crooked or Circular Lines are those which have a possibility of diducting or setting farther asunder their extreams, as the Line A⌒B

2. Streight or Right Lines are thoſe which have no poſſibility of diducting or ſetting farther aſunder their extreams, and therefore is the leaſt Length between the two extreams; *Plato* defines a right line to be that whoſe extreams doth ſhadow all the middle parts, and is repreſented by the Line B C.

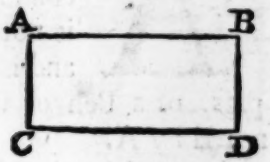


3. Mixt Lines are compounded of Streight and Circular Lines, as the Line



III. A Superficies is that which hath Length and Breadth, the thickneſs not being conſidered; and as a Line is produced by the Motion of a point, ſo a Superficies is produced by the Motion of a Line ſuppoſed to move tranſverſly, that is to ſay, the Line A C being ſuppoſed to move ſide-ways to B D it will produce a Superficies A B C D.

IV. The extreams of a Superficies are Lines. But of a Circular Superficies a Line is the extream.



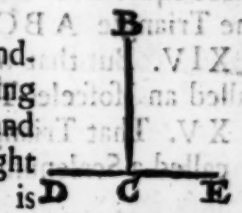
V. An Angle is an inclination of two Lines, the one to the other, the one touching the other, and not lying ſtreight forth at length. And of Angles there are three ſorts, namely Right-lined, Curve-lined and Mixt.

Note. When an Angle is mentioned by three Letters, the middlemoſt Letter ſignifies the Angle intended.

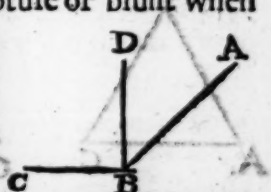
So

So ABC is a right lined Angle, DEF is a Curve lined Angle, and GHI is a mixt Angle.

VI. When a right Line BC ſtanding upon a right Line DE, making the Angles on either ſide BCD, and BCE equal; they are called right Angles, and the right Line BC is called a perpendicular Line to that upon which it is erected, *viz.* DE.



VII. An Angle is ſaid to be obtuſe or blunt when it is greater than a right Angle; and acute or ſharp when it is leſs than a right Angle: So the Angle ABC is an obtuſe Angle, and the Angle ABD is an acute Angle.



VIII. A Circle is a plain Figure, comprehended by one Line, being generated by the Motion of a Compaſs or other equivalent means, wherein all right Lines drawn from the Centre to the circumference are of equal length; the Centre is a Point exactly in the middle of the Circle.

IX. The Diameter of a Circle is a right Line, as AB drawn by the Centre C, and being terminated by the circumference on either ſide, divides the Circle into two equal Parts.

X. The Semi-diameter is half the Diameter, as AC or CB.

XI. A Semicircle is one half of the whole Circle.

XII. Trilateral or three sided Figures are those which are contained within three right Lines, and are called Triangles, because three Lines being joyned together at Angles, constitute three Angles.

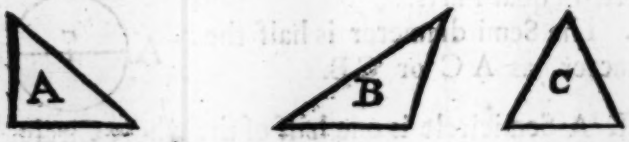
XIII. Of three sided Figures, that which hath three equal sides is called an Equilateral Triangle, as the Triangle ABC.

XIV. But that which hath two sides alike equal, is called an Isosceles Triangle, as CDE.

XV. That Triangle whose three sides are unequal, is called a Scalenum, as EFG.

Equilateral. *Isosceles.* *Scalenum.*

XVI. There are also other Triangles, and are named according to the Quality of their Angles. A right angled Triangle, is that which hath one right Angle, as the Triangle A. An Amblygonium or obtuse-angled Triangle, is that which hath one Angle obtuse, as the Triangle B. An Oxigonium or acute angled Triangle, is that which hath three acute Angles, as the Triangle C.



XVII. In

XVII. In every Triangle, two of the Lines being taken for two ſides, the third Line remaining is called the Baſe, whether it be the lowermoſt Line of the Triangle or not, ſo that any one of the three Lines which incloſe a Triangle may be taken for the Baſe.

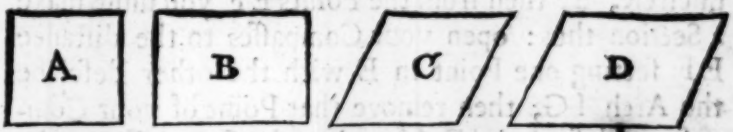
XVIII. Of Quadrilateral or four-ſided Figures, there are ſeveral ſorts.

XIX. A Quadrate or Square, is that which hath equal ſides and right Angles, as the Figure A.

XX. An Oblong hath the oppoſite ſides alike, and right Angles, as the Figure B.

XXI. A Rhombus or Diamond Figure hath four equal ſides, and two Angles acute, and two obtuſe, as C.

XXII. A Rhomboides hath oppoſite ſides, and oppoſite Angles alike, but it is neither equilateral nor right Angled, as the Figure D.



XXIII. A Parallelogram is a four-ſided Figure, whoſe oppoſite ſides are Parallel, as the Figure E.

XXIV. All other four-ſided Figures are called Trapezia's or Tables, as F.

XXV. Parallel Lines are ſuch, which being in the ſame ſuperficies, if produced will not meet, as G.

Thus much may ſerve for Definition; I proceed now to Demonſtration and Practice.

CHAP. II.

Note, *That the Figures belonging to the following Propositions are in the next folded page,*

PROP. I. Fig. I.

From a Point given in a ſtreight Line to raiſe a Perpendicular that ſhall cut the ſtreight Line at right Angles.

LEt C be the Point propoſed in the Line AB, from which a Perpendicular is to be raiſed.

From the Point given C draw at pleaſure any Semicircle, EF then from the Points E F you muſt make a Section thus: open your Compaſſes to the diſtance E F ſetting one Point in E with the other deſcribe the Arch I G, then remove that Point of your Compaſs, and ſet it in F (keeping the Compaſſes at the ſame diſtance) with the other Point thereof deſcribe the Arch I H which will interſect or cut through the Arch I G, from which Interſection at I, a Line being drawn to the point C, will be Perpendicular, and cut the Line A B at right Angles.

This Prop. *may be performed, as in* Fig. II.

By opening your Compaſs to any convenient diſtance, and ſetting one of the Points of the Compaſſes in the Point C, with the other Point make the prick A, then turning the Compaſſes, keeping one Point ſtill at C,

C, with the other Point make the prick B in the Line given, then ſetting one Foot in A, and opening the Compaſſes to B, deſcribe the Arch H I, alſo remove the Compaſſes, and ſetting one Point in B, deſcribe another Arch H K, from which Interſection draw the Line H C, which will cut the Line A B at right Angles.

PROP. II. Fig. III.

To raiſe a Perpendicular at the end of a Line.

LEt the Line given be A B, and on the end B it is required to raiſe a Perpendicular.

Set one Foot of your Compaſſes above the Line A B at pleaſure, as ſuppoſe at D, and opening the other Point till it ſtay in B, with this diſtance keeping the Point at D, deſcribe a Semicircle F B E, by the Points F and D draw the ſtreight Line F D E, and where that ſtreight Line cuts the Circle as at E, from thence draw the Perpendicular E B.

To perform this Prop. *another way.* Fig. IV.

LEt the point B at the end of the Line A B be the point from whence a Perpendicular is to be raiſed.

The Compaſſes being opened at pleaſure, ſet one Foot in the Point B, and with the other Foot deſcribe the Arch of a Circle C D G, keeping the Compaſſes at the ſame diſtance, ſetting one Foot in C deſcribe the Arch B D, then ſetting one Foot in D, deſcribe the Arch F E B and from the Point E draw the Arch I D, and from the interſection of this Arch with the Arch F E B draw the Perpendicular I B.

PROP.

PROP. III. Fig. V.

To raiſe a Perpendicular at the end of a Line having little or no Paper beyond that end of the Line whereon you are to raiſe the Perpendicular.

V: Adams 56

LET AB be the Line given and B the end. Opening the Compaſſes at pleaſure, ſetting one Foot in B, with the other Foot deſcribe the Arch CD, then remove one Foot of the Compaſs, and ſet it in C, and interſect the Arch CD at E, then lay a Ruler from C to E and ſtrike a Line as EF, then ſet one Foot of your Compaſſes (being at their firſt diſtance) in E, and with the other Foot make a Point or Prick in the Line EF at F, from whence draw the Perpendicular FB.

PROP. IV. Fig. VI.

Upon an Angle given to raiſe a Perpendicular.

LET ABC be the Angle given; ſetting one Foot of your Compaſſes in B deſcribe the Arch AC, then opening the Compaſſes a little wider, and ſetting one Point in C deſcribe the Arch DE, then removing the Point to A, interſect the Arch DE at F, from whence draw the Perpendicular FB.

PROP.

PROP. V. Fig. VII.

To let fall a Perpendicular upon a Line given, and from a Point above the Line.

LEt the Line given be AB, and the Point given be C; ſetting one Foot of the Compaſſes in C deſcribe the Arch DE and from D and E make the interſection at F from whence draw the Line CF.

PROP. VI. Fig. VIII.

By a Point given to draw a Line parallel to a right Line given.

LEt A be the Point by which we muſt draw a Line which may be parallel to the Line BC.

Draw at pleaſure the Oblique (or Diagonal) Line AD, from the Point A draw the Arch DE, and from the Point D deſcribe the Arch AF, then ſetting one Point of the Compaſſes in the given Point A, extend the other Point to that part of the given Line which is interſected by the Arch AF, with this diſtance, ſetting one Point of the Compaſſes at the Interſection D, extend the other upon the Arch DE, and where that Point falls upon the Arch as at G, draw a Line from the Point A through the Point at G, and it will be parallel to the Line BC.

Another

Another way to draw a Parallel to the Line BC. Fig. IX.

The Compaſſes being ſet to the ſpace you intend ſhall be between the two Lines (otherwiſe any diſtance will ſerve) ſetting one Foot on the end of the Line B, with the other Foot deſcribe the Arch DE, this being done, keeping the Compaſſes at the ſame diſtance, ſet one Point on the end of the Line C, and deſcribe the Arch FG, then draw the Line HI, juſt to touch the uppermoſt part of theſe two Arches, and it will be parallel to the Line BC.

PROP. VII. Fig. X.

To cut a right Line AB *equally in the middle.*

THe Compaſſes being opened to any diſtance ſhorter than the whole Line, and longer than half the Line; as ſuppoſe them opened from A to C, ſetting one Point of the Compaſs in A, with the other deſcribe the Arch DE, keeping the Compaſſes at the ſame diſtance ſetting one Foot in B, with the other deſcribe the Arch FG, and from the Interſection (or cutting through) of theſe two Arches, draw the Line HI, it will divide the Line AB equally in the middle.

The Figures of the Propoſitions following, you will find in the next folded Page to this.

PROP.

Fig. I.

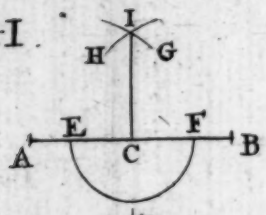


Fig. II.

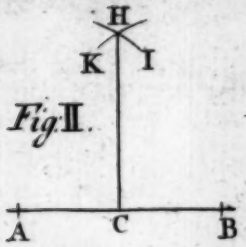


Fig. III.

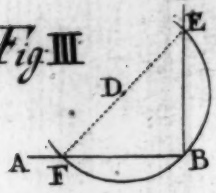


Fig. IIII.

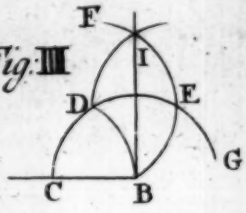


Fig. V.

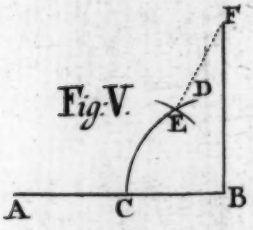


Fig. VI.

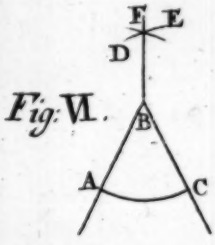


Fig. VII.

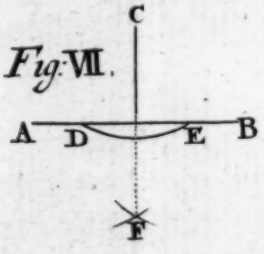


Fig. VIII.

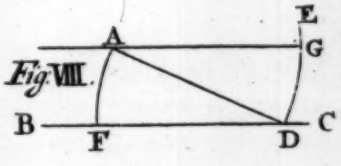


Fig. IX.

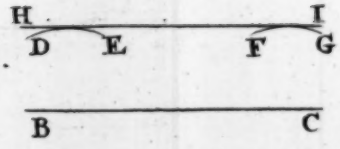
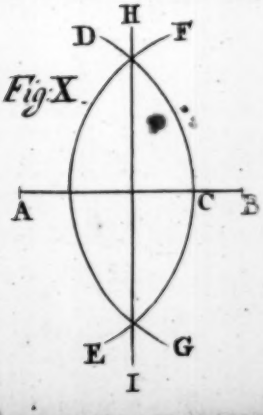


Fig. X.



UMI

PROP. VIII. Fig. XI.

To cut a right Lined Angle BAC *into two equal Parts.*

OPening the Compaſſes at pleaſure (*viz.* opening them to any diſtance) ſetting one Point of them on the Angle A, with the other Point deſcribe the Arch DE, then from D and E make the Interſection at G, through which Interſection and the Angle A, draw the Line AG, which divides the Angle BAC into two equal parts.

PROP. IX. Fig. XII.

To make a Triangle ABC *of three right Lines,* (viz. AB, BC, CA,) *equal to three Lines given* D. E. F. *of which three Lines any two being taken and added together muſt be longer than the third Line remaining; otherwiſe you cannot make a Triangle of them.*

FIrſt draw the ſtreight Line GH, then upon that Line take GA equal to the given Line D; alſo upon the ſame Line GH ſet the Line E from A to C; alſo ſet the given Line F from C to D, then opening the Compaſſes to GA, with one Point in A deſcribe the Circumference GBC, this being done, ſet

ſet one Point of the Compaſs at C, and the other being opened to D, deſcribe the circumference DBI, then joyn the Triangle where theſe two Circles interſect which is at B, and make the Triangle ABC, which is equal to the three Lines given D, E, F.

PROP. X. Fig. XIII.

At a Point A *in a right Line given* AB, *to make a right Lined Angle* A, *equal to a right Lined Angle given* D.

SEt one Point of the Compaſſes at D, and with the other deſcribe the Arch EF, then with the Compaſſes being at the ſame diſtance, ſetting one Point at A, with the other Point deſcribe the Arch BC, then take the Chord Line of the Arch EF (*viz.* the ſtreight Line EF) between the Points of your Compaſſes, and ſet one Point at B, and where the other Point toucheth the Arch BC as at C, by that Point draw the Line AC, and the Angle A, will be equal to the Angle D.

PROP. XI. Fig. XIV.

To make a Parallelogram ABCD *equal to a Triangle given* EFB *in an Angle equal to a right Lined Angle given* G.

THrough the Point E draw the Line ED parallel to the Line FB (by the 6. Prop.) then upon the Point B raiſe the Line BD making the Angle at

at B equal to the given Angle G, (as you were taught in the 10. Prop.) then Biſect (*viz.* divide in the middle) the Baſe FB as at A, and draw the Line AC parallel to BD, and the Parallelogram ABCD will be equal to the Triangle given EFB, and like the Angle given G.

PROP. XII. Fig. XV.

Upon a right Line given A, *to make a Parallelogram* FL *at a right Lined Angle given* C, *equal to a Triangle given* B.

BY the foregoing Prop. make a Parallelogram FD equal to the Triangle B, ſo that the Angle GFE may be equal to the Angle given C; continue the Line GF till FH be produced equal to the given Line A; then by the Point H draw the Line IL parallel to EF, alſo continue the Line DE till it touch the Line HI, then draw the Diagonal Line IK till it meet with the Line DG being continued, then through the Point K draw the Line KL parallel to GH, then extend or continue the Line EF unto M, and the Line IH unto L, then ſhall FL be the Parallelogram required, for the Parallelogram FL is equal to the Parallelogram FD, and FD is equal to the Triangle given B, and the Angle MFH is equal to GFE, and GFE is equal to the given Angle C.

PROP.

Prop. XIII. Fig. XVI.

Parallelograms BCDA, GHFE *ſtanding upon equal Baſes* BC, GH, *and betwixt the ſame Parallels* AF, BH *are equal one to the other.*

DRaw BE and CF, becauſe BC is equal to GH, and GH equal to EF, therefore is BCFE a Parallelogram. Whence the Parallelogram BCDA is equal to BCFE and that equal to GHFE which was to be Demonſtrated.

Prop. XIV. Fig. XVII.

Triangles BCA, BCD *ſtanding upon the ſame Baſe* BC, *and between the ſame Parallels* BC, EF, *are equal one to the other.*

DRaw BE parallel to CA, and CF parallel to BD, then is the Triangle BCA equal to half the Parallelogram BCAE, and alſo equal to half BDFC and that equal to the Triangle BCD which was to be Demonſtrated.

Prop.

PROP. XV. Fig. XVIII.

If a Parallelogram ABCD *have the ſame Baſe* BC *with the Triangle* BCE, *and be between the ſame Parallels* AE, BC, *then is the Parallelogram* ABCD *double to the Triangle* BCE.

LEt the Line AC be drawn. Then is the Triangle BCA equal to BCE. Therefore is the Parallelogram ABCD equal to two ſuch Triangles as BCA, and likewiſe alſo equal to two ſuch Triangles as BCE, which was to be Demonſtrated.

From hence may the Area (or Content) of any Triangle BCE be found. For whereas the Area of the Parallelogram ABCD is produced by the Altitude drawn into the Baſe, therefore ſhall the Area of a Triangle be produced by half of the Altitude drawn into its Baſe, or half its Baſe drawn into its Altitude; as if ſo be the Baſe BC be 8, and the Altitude 7, taking the half of the Baſe 4, and multiplying it by the Altitude 7, it produceth 28, which is the Area or Content of the Triangle BCE; or otherwiſe if you take the whole Baſe 8, and half the Altitude 3 and an half, and multiply them, they produce 28 (as before) for the Content.

Prop. XVI. Fig. XIX.

Upon a right Line given FG, *to make a Parallelogram* FL *equal to a right Lined Figure given* ABCD, *at a right Lined Angle given* E.

REſolve the right Lined Figure given into two Triangles BAD, BCD, then make a Parallelogram FH equal to BAD, ſo that the Angle F may be equal to the Angle E; (as you were taught at the XII. Prop.) FI being produced to K, make the Parallelogram IL equal to the Triangle BCD. Then is the Parallelogram FL equal to FH more IL, and therefore equal to the Figure given ABCD which was to be done.

Schol. Fig. XX.

HEnce is eaſily found the the exceſs HE, whereby any right Lined Figure A, exceeds a leſs right Lined Figure B; namely, if to ſome right Line CD both be applyed, and both the Trapezia's each of them being divided into two Triangles, and working as before is taught, you will find the Parallelogram DF equal to the Trapezia A, and the Parallelogram DH equal to the Trapezia B, ſo that the Figure A exceeds the Figure B by ſo much as the Parallelogram GEFH contains.

Prop.

PROP. XVII. Fig. XXI.

In right Angled Triangles BAC, *the Square* BE, *which is made of the ſide* BC *that ſubtends the right Angle* BAC, *is equal to both the Squares* BG *and* CH *which are made of the ſides* AB, AC, *containing the right Angle.*

JOin AE and AD, and draw AM Parallel to CE, becauſe the Angle DBC is equal to FBA, add the Angle ABC common to them both, then is the Angle ABD equal to FBC. Moreover AB is equal to FB, and BD equal to BC; therefore is the Triangle ABD equal to FBC. But the Parallelogram BM is equal to two ſuch Triangles as ABD, and the Parallelogram or Square BG is equal to two ſuch Triangles as FBC (for GAC is one right Line by the Hypotheſis) therefore is the Parallelogram BM equal to the Quadrate BG. By the ſame way of argument is the Parallelogram CM equal to the Quadrate CH; therefore is the whole Parallelogram (or Square) BDEC equal to the two Quadrates BAGF and ACIH, which was to be done.

PROP. XVIII. Fig. XXII.

There are three Quadrates or Squares given, whereof the ſides are AB, BC, CE, *and it is required to make one Square, whoſe Area or Content ſhall be equal to the Area of thoſe three Squares.*

MAke the right Angle FBZ, having the ſides infinite (the meaning of infinite is to draw the ſides long enough, and of what length there is no determination) And on theſe two ſides transfer AB and BC; that is to ſay, take the given Line AB between your Compaſſes, and place it from the Angle B to A; alſo take the Line given BC, and place it from the Angle B to C; joyn AC (*viz.* draw the Line AC) then is a Square whoſe ſide is AC, equal to two Squares made of the two Lines AB and BC, then take the Line AC and place it from B to X; alſo take the third given Line or ſide CE, and place it from B to E, then draw the Line EX, then a Square being made whoſe ſide is EX, is equal to three Squares being made of the three Lines or ſides given AB, BC, CE, which was to be done. The truth whereof is manifeſted by Arithmetick; for let the Line AB be 8 Feet in length, then the Line BC will be 5 Feet, and the Line CE will be 4 Feet; and the Line EX will be 10 Feet and 6 Inches: Now the Square of 8 is 64, and the Square of 5 is 25, and the Square of 4 is 16, which three Squares being added

ded together produce 105, for the Area of the three given Squares; therefore the Square of the Line EX being 10 Feet and 6 Inches, is 105 Feet, and equal to the three Squares given.

Prop. XIX. Fig. XXIII.

Two unequal right Lines being given AB, BC; *to make a Square equal to the difference of the two Squares of the given Lines* AB, BC.

FRom the Centre B, with the diſtance BA, deſcribe a Semi-circle, and from the Point C erect a Perpendicular CE meeting with the Circumference in E, and draw BE. Then is the Square of BE (or BA) equal to the Square of BC and CE. Therefore when the Square of BC is taken out of the Square of BA, the remaining part of the Square of BA will be equal to the Square of CE, which was to be done.

Prop. XX. Fig. XXIV.

Any two ſides of a right Angled Triangle ABC *being known, to findout the third.*

LEt the ſides AB, AC, encompaſſing the right Angle, be the one 6 Feet, the other 8 Feet; Therefore whereas the Square of AB is 36, and the Square of AC is 64 which being added make 100, therefore (as you were taught at the 17 Prop.) the

other ſide ſought for muſt be equal in power (*viz.* being Squared) to the two given ſides being Squared, which contain 100, whoſe Square Root is 10 the length of the ſide ſought B C; which was to be done.

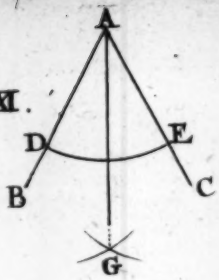
PROP. XXI. Fig I.

To deſcribe a Circumference that ſhall touch any three Points given, provided they are not in a right Line; ſuppoſe the Points given to be A. B. C. *(the Figures of this and the following Propoſitions you will find in the next folded Page.)*

TAke the diſtance between A and B between your Compaſſes, and ſetting one Point of the Compaſs on the Point by A deſcribe the Arch D E, then with the ſame diſtance ſetting one Point of the Compaſs on the Point by B, deſcribe another Arch which will cut the former Arch in the Points by 1, then laying a Ruler to the Points by 1, where the Arches interſect, draw a ſtraight Line F, G, this being done, take with your Compaſſes the diſtance between the other Point C and B, and with this diſtance ſetting one Point in B, deſcribe the Arch H, I, then with the Compaſſes at the ſame diſtance, ſetting one Point on the Prick by C, deſcribe another Arch cutting the former in the Points by 2, through which Points draw another ſtreight Line till it cut through the firſt ſtreight Line, as at G; I ſay G is the Centre from whence the circumference A. B. C is deſcribed, which toucheth the three given Points.

PROP.

Fig: XI.



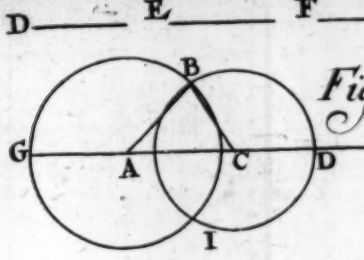
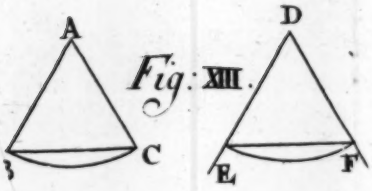


Fig: XIII.



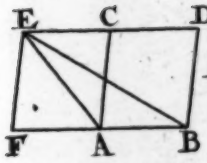


Fig: XV

Fig: XIV.

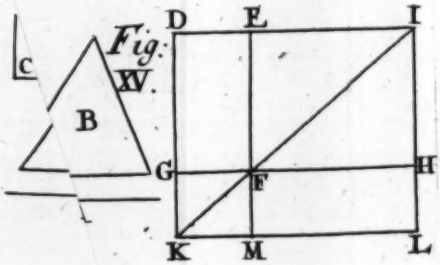


Fig: XVI.

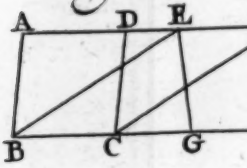


Fig: XVII.

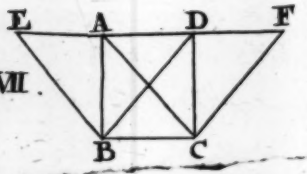


Fig: XVIII.

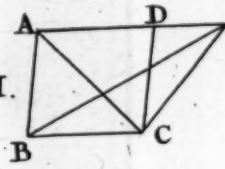


Fig: XIX.

Fig: XX

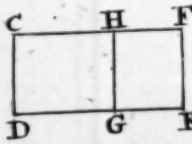


Fig: XII

Fig: XIV

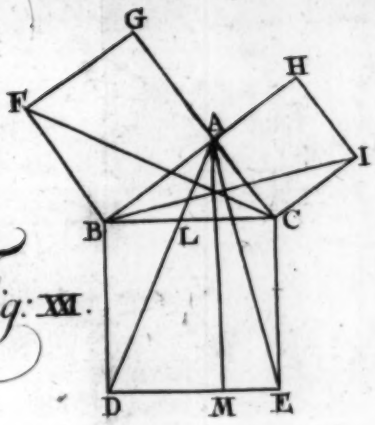


Fig: XXI.

Fig: XXII.

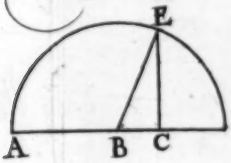


Fig: XXIII.

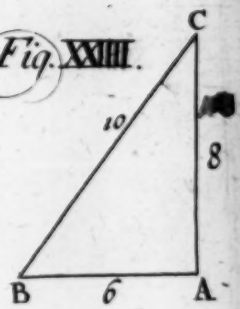


Fig. XXIIII.

Pag: 20.

PROP. XXII. Fig. II.

To deſcribe an Oval upon a length given A B.

DIvide the given Line into three equal parts (as the next Propoſition following will teach) A C D B, ſetting one Point of the Compaſſes at the Point by C, with the diſtance C A deſcribe the Circle A E F, then with the ſame diſtance ſetting one Point by D deſcribe the Circle B E F, then draw four ſtreight Lines through the Centres C and D, and the Interſection of the two Circles E and F; then ſetting one Point of the Compaſſes in E, and extending the other Point to I, deſcribe the Arch I H, then with the Compaſſes at the ſame diſtance, ſetting one Point on the Interſection by F, deſcribe the Arch O P, which concludes the Oval. *Note*, that I A O and H B P, are vulgarly called Hanſes, and I H and O P, are called Scheams.

PROP. XXIII. Fig. III.

To divide a ſtreight Line given A B; *into three equal parts.*

FRom the end A, draw at pleaſure the Line A C making what Angle you will, then from the other end of the Line B draw the Line B D parallel to the Line A C, then opening your Compaſſes at pleaſure, ſetting one Point in A, turn them three times over the Line A C, which will make three Di-

viſions, *viz.* E F G, then with the Compaſſes continuing at the ſame diſtance, ſetting one Foot in B, make three Diviſions on the Line B D, *viz.* H I K, then draw with a Ruler and the Point of the Compaſs a ſtreight Line from A to K, another from E to I, alſo another from F to H, and another from G to B, and they being drawn, the given Line A B is divided into three equal parts; which was to be done. You may if you pleaſe divide the ſame Line or any other, into what number of equal parts you pleaſe, by dividing the two parallel Lines A C and B D, into ſo many equal parts as you would have the given Line divided into.

Prop. XXIV. Fig. IV.

To deſcribe an Oval equal in length to the firſt Oval not riſing ſo high.

Divide the given Line A O into four equal parts (by the foregoing Prop.) in B C D, then taking one of thoſe parts between the Compaſſes; upon the Centres B C D, deſcribe three Circles; and thoſe two parts of the middlemoſt Circle that is without the two other Circles, divide in the middle at E and F, then from E to the Centre D, draw a ſtreight Line, and continue it to the circumference at 4, alſo draw another ſtreight Line from E through the Centre by B to the circumference, which will cut it at 3; Likewiſe from F through the ſame Centres draw right Lines which will cut the two circumferences, the one in 2, the other in 1. Then from the Centre F with the Radius (or diſtance) F, 1, deſcribe the Arch 1, 2, alſo from the Centre E with the ſame Radius deſcribe the Arch 3, 4, which concludes the Oval.

Prop.

PROP. XXV. Fig. V.

Another way to deſcribe Ovals.

UPon the Line given deſcribe two Equilateral Triangles, joyn them together with one Common Baſe, ſo that they make a Rhombus; then continue (or draw) the Line A C to 3, ſo that C 3, may be ſix ſuch parts whereof A C is five, *viz.* It muſt be the length of A C, and one fifth part more of it; alſo draw the Line B D to 2, that it may be the ſame length with A 3, alſo draw B C to 1, and A D to 4, being all of one length: Then from the Centre A with the Radius A 3, deſcribe the Arch 3, 4, alſo from the Centre B, with the ſame Radius deſcribe the Arch 1, 2, then from the Centre C, with the Radius C 1, deſcribe the Arch 1, 3; likewiſe from the Centre D, with the ſame Radius deſcribe the Arch 2, 4, which will encloſe the Oval; From theſe four Centres you may deſcribe Ovals, greater or leſſer as you pleaſe.

PROP. XXVI. Fig. VI.

To deſcribe an Oval according to any length and breadth given.

LEt the length given be A B, and the breadth C D.

Apply the two given Lines together, ſo that they may cut each other into two equal parts, and at right Angles in the Point E, then take half the Line A B between

between your Compaſſes, and ſetting one Point of the Compaſſes in C, extend the other till it touch the Line A B in K and L, which two Points are called the burning Points, in which Points drive two Nails if you deſcribe it upon Boards, but upon Paper as here, two Pins will do; the Pins being ſtuck firm in the Points K and L, ſtick alſo another Pin in the Point C, then take a Thread and encompaſs theſe three Pins in form of a Triangle, pulling the Thread tite, tye the two ends of the Thread together by a knot at C, then taking out the Pin at C, take a Pencil, holding it cloſe to the inſide of the Thread, and carrying the Pencil round upon the Paper, about the Pins with the Thread always ſtreight, the Ellipſis or Oval A C B D ſhall be thereby deſcribed.

Prop. XXVII. Fig. VII.

To find the Centre and the two Diameters of an Oval.

Let S E T D be the Oval whereof the Centre and the Diameters are to be found.

Within the Oval, draw at diſcretion the Parallel Lines E F, G H; cut theſe Lines into two equally in I and K, draw the Line I K, cut it into two equally in L, which is the middle Centre of the Oval; upon this Centre L, deſcribe at pleaſure the Circle M N O, cutting the Oval in P and Q, from which Sections draw the right Line P Q, cut it in the middle in R, from which, through the Centre L, draw the great Diameter S T, and from the Centre L, draw the leſſer Diameter E L D parallel to the Line P Q, which was to be done.

Prop.

PROP. XXVIII. Fig. VIII.

To deſcribe an Oval with a pair of Compaſſes to any length and breadth given.

I ſhall only deſcribe a Semi-oval, and according to the ſame rules, if you will, you may deſcribe the whole Oval.

Let the length given be A B, and one half of the breadth C D; divide A B into ſeven equal parts, then upon one ſeventh part from A, as at E, raiſe a Perpendicular from the Line A B (*viz.* E G). Alſo at one ſeventh part from B, as at F, raiſe another Perpendicular F H; then divide the half breadth given C D into fifteen equal parts, and take eleven of thoſe parts and ſet upon the Perpendicular from E to G, and likewiſe from F to H; then taking the ſpace between A and G, ſetting one Point of the Compaſſes in A, deſcribe the Arch G i; keeping the Compaſſes at the ſame diſtance, ſet one Point in G, and deſcribe another Arch which will cut the former in the Point by i, from which Point with the Radius A G, deſcribe the Semi-hans A G; This being done, take between your Compaſſes the ſpace B H, and ſetting one Point in B, deſcribe the Arch I, i, then remove your Compaſſes to H, and Interſect that Arch in the Point by i, then ſetting your Compaſſes on the Point i, with the ſame diſtance deſcribe a part of the Oval B H, which part, as alſo the other part A G, are vulgarly called Semi-hanſes, becauſe it is but a Semi-oval (Semi, ſignifies half) but if had been an whole Oval, then the Semi-hanſe above the Line

A,

A, and another Semi-hanſe below the Line A, being joined, is called an Hanſe from the Latin word *Hanus* ſignifying a great Bellied thing; The other part to be deſcribed from G to H is called the Scheam, which to deſcribe, continue or draw longer the half breadth DC, and in that Line find a Centre, whereon ſetting one Point of the Compaſſes, the other Point may touch the three Points G, D, H, as on the Centre I, whereby deſcribe the Scheam G D H, which was to be done.

CHAP. III.

A Digreſſion concerning Ellipſis Arches.

AND ſince Ellipſis or Semi-oval Arches being neatly wrought in Brick, ſhew very pleaſant; and are ſometimes uſed over Gate-ways, and ſometimes over Kitchin Chimnys inſtead of Mantle-trees, I think fit to write ſomething concerning them relating to Brick-layers, making the Moulds, and dividing the Courſes.

The Ellipſis you may deſcribe to what length and height you pleaſe, either by the laſt Prop. or by the 26*th*.

We will ſuppoſe an Ellipſis Arch to be made over a Chimney, whoſe Diameter between the Jaums is 8 Feet, and the under ſide of the Arch at the Key to riſe in height 18 Inches from the level of the place whence you begin to ſpring the Arch; The height or depth of the Arch we will ſuppoſe to be made of the

Fig: I.

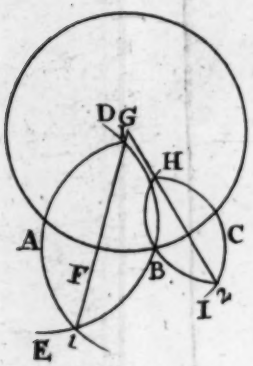


Fig: II.

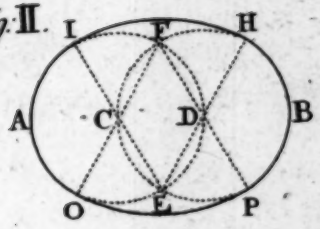


Fig: III.

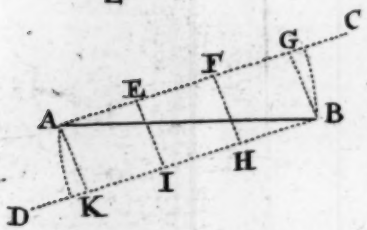
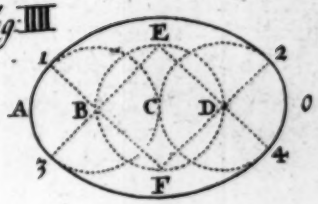


Fig: IIII



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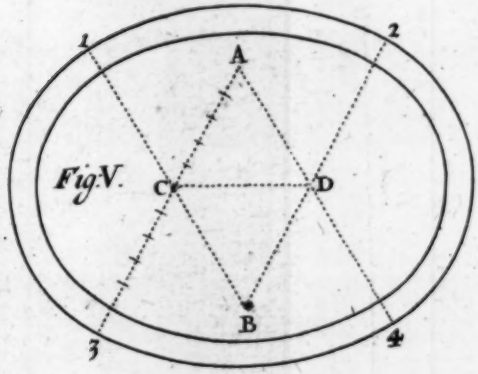


Fig: VI

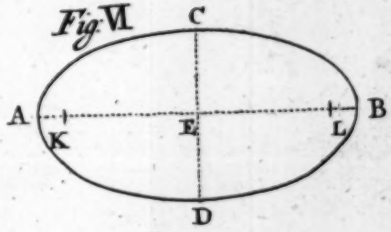
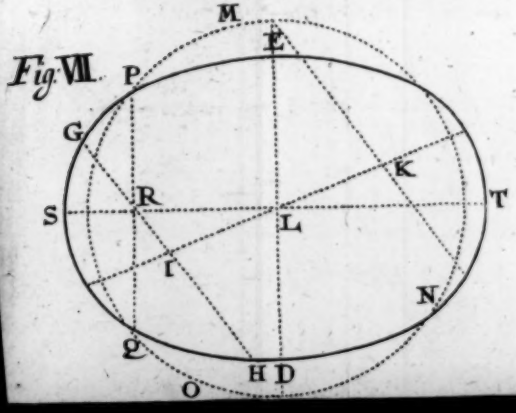


Fig: VII.



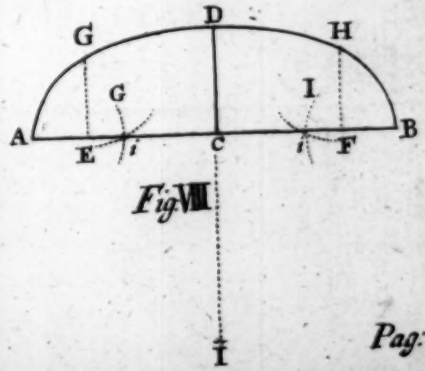


Fig: VIII

the length of two Bricks, which when they are cut to the ſweep of the Arch, will not contain above 14 Inches, and perhaps you muſt Cement pieces to many of the Courſes in the Hanſe to make them long enough to contain or hold 14 Inches, eſpecially if you intend to make the Courſes of the Hanſe, and the Courſes of the Scheam to ſeem alike in greatneſs on the under ſide of the Arch. For if you make the Hanſe to come to a true Sommering for the Scheam by that time that you have ended the Hanſe, and are ready to ſet the firſt Courſes of the Scheam: The Mould, and ſo likewiſe each Courſe in the Hanſe, will be much leſs at the lower part or under ſide of the Arch, then the Mould or Courſes of the Scheam, as you may perceive by the Hanſe BK, in the IX. *Fig.* which way of working theſe kind of Arches is ſtronger, than to make the Courſes ſeem alike in bigneſs in Hanſe and Scheam, although it be not ſo pleaſing to the eye: In the IX. *Fig.* I will ſhew how to make one half of the Arch this way, and in the other half ſhew how to make the Courſes in Hanſe and Scheam of a bigneſs.

Firſt, Deſcribe the under ſide of the Arch (*viz.* the Ellipſis A D B, whoſe Diameter A B is 8 Feet, and the height C D 18 Inches) upon ſome ſmooth Floor or ſtreight plaiſtered Wall or ſuch like; then continue (*viz.* draw longer) both the Lines A B, C D, cutting each other at right Angles, then from A to E; alſo from B to F, likewiſe from D to G, ſet 14 Inches the intended height of your Arch. Then deſcribe another Ellipſis to that length and height after this manner: Lay a ſtreight Ruler on the Centre by I, and on the joyning of the Hanſe and the Scheam together, as at K, and draw the Line K L, then ſet

one

one Point of your Compaſſes in the Centre of the Hanſe at M, and open the other Point of the Compaſſes to F, and deſcribe the upper Hanſe F L, likewiſe ſetting one Point of the Compaſſes in the Centre by I, with the other extended to G, deſcribe the Scheam G L; (although I ſpeak here of Compaſſes, yet when you deſcribe an Arch to its full bigneſs, you muſt make uſe of Centre Lines or Rules; the laſt are beſt, becauſe Lines are ſubject to ſtretch) then taking between your Compaſſes the thickneſs of a Brick, abating ſome ſmall matter which will be rub'd off from both beds of the Brick; with the Compaſſes at this diſtance divide the upper Hanſe from L to F into equal parts, and if they happen not to divide it into equal parts, then open them a ſmall matter wider, or ſhut them a ſmall matter cloſer, till it doth divide it into equal parts, and look how many equal parts you divide the upper Hanſe into, ſo many equal parts you muſt divide the lower Hanſe from K to B into likewiſe (or you may divide the upper Hanſe from the Centre O, making a right Angle from each Sommering Line to the Ellipſis, as is ſhewn in deſcribing the ſtreight Arches following; and from the Centre O, and the Diviſions in the upper Hanſe being thus divided, you may draw the ſtreight Lines to the lower Hanſe, and not divide it with the Compaſſes) through each of which Diviſions with a Rule and Pencil draw ſtreight Lines, then get a piece of thin Wainſcot, and make it to fit between two of theſe Lines, allowing what thickneſs for Morter you intend, this will be the Sommering Mould for the Hanſe; then divide the upper Scheam likewiſe, with the Compaſſes at the ſame diſtance into equal parts, and laying a Ruler on the Centre I, from each Diviſion in the Scheam G L,

draw

draw ſtreight Lines to the lower Scheam DK, then make another Sommering Mould to fit between two of theſe Lines, abating ſo much as you intend the thickneſs of your Joints of Mortar to be, which if you ſet very cloſe Morters, the breadth of the Line will be enough to allow; then laying the Inner edge of a Bevil ſtreight on the Line K L, bring the Tongue to touch the under ſide of the firſt Courſe of the Scheam, then take up the Bevil, and ſet that Bevil Line upon the Sommering Mould of the Scheam, which Bevil Line ſerves for each Courſe in the Scheam: but you muſt take the Bevil of each Courſe in the Hanſe, and ſet them upon your Sommering Mould, and number them with 1, 2, 3, 4, *&c.* becauſe each Courſe varies.

Thus having made your Sommering Moulds, in the next place you muſt make the Moulds for the length of your Stretchers, and for the breadth of the Headers and the Cloſiers; A piece of Wainſcot 7 Inches long, and 3 Inches and an half broad will ſerve for the length of the Stretchers, and the breadth of the Headers, the Cloſiers will be 1 Inch and $\frac{3}{4}$ broad.

So the Cloſier will be half the breadth of the Header, and the Header half the length of the Stretcher, which will look well.

It remains now to ſpeak ſomething to the other part of the Arch, to wit, A D, whoſe Courſes both in Hanſe and Scheam run alike upon the Ellipſis Lines, and ſeem of one bigneſs (although perhaps there may be ſome ſmall matter of difference, by reaſon I have not divided the Courſes in this Figure from a right Angle, but every Courſe from the Angle, which it makes with the Ellipſis, which I choſe rather to do that ſo the Bevil of one Courſe might not ſeem to

run

run more upon the Ellipſis than the Bevil of another, and the difference of the thickneſſes being ſo inconſiderate is not diſcerned.)

Having deſcribed both the Ellipſis Lines A D, E G, divide each of them into a like number of equal parts, always remembring to make each Diviſion on the upper Ellipſis Line no greater than the thickneſs of the Brick will contain when it is wrought; then through each Diviſion in both the Ellipſes draw ſtreight Lines, continuing them 4 or 5 Inches above the upper Ellipſis Line, and as much below the the lower Ellipſis Line, then having provided ſome thin Sheets of fine Paſtboard about 20 Inches ſquare, cutting one edge ſtreight, take one Sheet and lay the ſtreight edge even upon the Line A E, ſo that it may cover both the Ellipſis Lines, and being cut to advantage, it may cover 8 Courſes (or 9 of the ſtreight Lines) having laid it thus upon the Figure of the Arch, ſtick a Pin or two through it to keep it in its place, then lay a Ruler upon the Paſtboard to the 7, 8, or 9*th.* ſtreight Line of the Arch, according as the Paſtboard be in bigneſs to cover them, and take a ſharp Pen-knife, laying the Ruler upon the Paſtboard true to the ſtreight Line (whoſe ends being continued longer than the Arch is deep, as I directed before, will be ſeen beyond the Paſtboard) and cut the Paſtboard true to the Line, then take another Sheet and join to it, and cut it as you did the firſt, and ſo continue till you have covered the Arch from A E juſt to the Line D G, ſticking Pins in each Sheet to keep them in the places where you lay them: Then deſcribe both the Ellipſis Lines upon the Paſtboard, from the ſame Centres and Radii that you deſcribed the Ellipſis's under the Paſtboard, and either divide the Ellipſis Lines with the Compaſſes

on

on the Paſtboard, or elſe draw Lines upon the Paſtboard from or by the ſtreight Lines underneath whoſe ends you ſee; but the ſurer way is to divide the Ellipſis's on the Paſtboard, and draw Lines through thoſe Diviſions as you did beneath the Paſtboard; then ſet 7 Inches, being the length of each Stretcher, from A towards E, and from D towards G, and deſcribe from the former Centres the Ellipſis *oo* through each other courſe on the Paſtboard, as you may ſee in the Fig. alſo ſet three Inches and an half, being the breadth of the Header from A towards E, and likewiſe from D towards G; alſo ſet the ſame 3 Inches and an half from E towards A, and from G towards D, and deſcribe theſe two Ellipſis Lines from the ſame Centres through each Courſe which the Ellipſis Line of the Stretchers miſs'd; likewiſe draw in the ſame Courſes, two other Ellipſis Lines one Inch and $\frac{2}{4}$ from each of thoſe two Lines you drew laſt; which is the breadth of the Cloſiers; thus one Courſe of the Arch will be divided into two Stretchers, and the next to it into three Headers and two Cloſiers through the whole Arch; this being done, cut the Paſtboard according to the Lines into ſeveral Courſes, and each other Courſe into two Stretchers, and the Heading Courſe into three Headers, and two Cloſiers, exactly according to the ſweep of the black-lead Lines, and marke each Courſe with Figures, marking the firſt Courſe of the Hanſe with 1, the next with 2, the third with 3, and ſo continue till you have marked all the Courſes to the Key or middle, for every Courſe differs; you were beſt to mark the lower Cloſier in each Courſe with a Cipher on the left hand of its own number, that you may know it readily from the upper Cloſier, and make no miſtakes when you come

to ſet them, alſo the middle Headers in each Courſe ſhould be marked beſides its own number, the thickneſs of the upper Header being eaſily diſcerned from the lower Header needs no marking beſides its own number; The croſs Joints, and likewiſe the under ſide and upper ſide of each Courſe muſt be cut Circular, as the Paſtboards which are your Moulds direct you.

If you will add a Keyſtone and Chaptrels to the Arch, as in the Figure; let the breadth of the upper part of the Keyſtone be the height of the Arch, *viz.* 14 Inches, and Sommer, from the Centre at I, then make your Chaptrels the ſame thickneſs that your lower part of the Keyſtone is, and let the Keyſtone break without the Arch, ſo much as you project or ſale over the Jaums with the Chaptrels.

Other kind of Circular Arches, as half Rounds and Scheams, being deſcribed from one Centre, are ſo plain and eaſy, that I need ſay nothing concerning them: But ſince ſtreight Arches are much uſed, and many Workmen know not the true way of deſcribing them, I ſhall write ſomething briefly concerning them.

Streight Arches are uſed generally over Windows and Doors, and according to the breadth of the Piers between the Windows, ſo ought the Skew-back or Sommering of the Arch to be; for if the Piers be of a good breadth, as 3 or 4 Bricks in length, then the ſtreight Arch may be deſcribed (as its vulgarly called) from the *Oxi*, which being but part of a word is taken from the word *Oxigonium*, ſignifying an Equilateral Triangle with three ſharp Angles; but if the Piers are ſmall, as ſometimes they are but the length of two Bricks, and ſometimes but one Brick and an half, then the breadth of the Window or more may be

be ſet down upon the middle Line for the Centre, which will give a leſs Skew-back or Sommering than the Centre from an Oxi. I will ſhew how to deſcribe them both ways, and firſt from the Oxi.

Suppoſe a ſtreight Arch 1 Brick and an half in height to be made over a Window, 4 Feet in width. [See *Fig.* X.] wherein one half of the Arch is deſcribed from the Oxi, and the other half from the width of the Window. Let the width of the Window be A B; taking the width between the Compaſſes, from A & B as two Centres, deſcribe the two Arches, Interſecting each other at P, (though I ſpeak here of Compaſſes, yet when you deſcribe the Arch to its full bigneſs, you muſt uſe a Ruler or a Line, ſcarce any Compaſſes being to be got large enough.) Then draw another Line above the Line A B, as the Line C D, being parallel to it, at ſuch a height as you intend your Arch to be, as in this *Fig.* at 12 Inches; but moſt commonly theſe ſort of Arches are but 11 Inches in the height, or thereabouts, which anſwers to 4 Courſes of Bricks, but you may make them more or leſs in height according as occaſion requires; then laying a Ruler on the Centre P, and on the end of the Line A, draw the Line A C, which is vulgarly called the Skew-back for the Arch.

The next thing to be done, is to divide thoſe two Lines A B and C D into ſo many Courſes as the thickneſs of a Brick will contain being one of them, which ſome do by dividing the upper Line into ſo many equal parts, and from thoſe parts, and from the Centre P, draw the Sommering Lines or Courſes; others divide both the upper and lower Line into ſo many equal parts, and make no uſe of a Centre, but draw the Courſes by a Ruler, being laid from the Diviſi-

ons on the upper Line, to the Diviſions on the lower Line, both which ways are falſe and erroneous; [but this by way of Caution.]

Having drawn the Skew-back A C, take between your Compaſſes the thickneſs that a Brick will contain, which I ſuppoſe to be two Inches when it is rub'd, and ſetting one Point of the Compaſſes on the Line C D, ſo that when you turn the other Point about it may juſt touch the Line A C in one place, and there make a Prick in the Line C D, but do not draw the Sommering Lines until you have gone over half the Arch, to ſee how you come to the Key or middle; and if you happen to come juſt to the middle Line, or want an Inch of it, then you may draw the Lines, but if not, then you muſt open or ſhut the Compaſſes a little till you do.

Then keeping one end of the Rule cloſe to the Centre at P, (the ſureſt way is to ſtrike a ſmall Nail in the Centre P, and keep the Rule cloſe to the Nail) lay the other end of the Rule cloſe to the Prick that you made on the Line C D, keeping the Compaſſes at the ſame width (*viz.* 2 Inches) ſet one Point of the Compaſſes on the Line C D as before, ſo that the other Point being turned about may juſt paſs by the Rule, and as it were touch it in one place (you muſt remove the Point of the Compaſſes upon the Line C D, farther or nearer to the Rule, until it juſt touch the Rule in one place) and ſo continue with the Rule and Compaſſes until you come to the middle Line, and if it happen that your laſt ſpace want an Inch of the middle, then the middle of the Key-courſe will be the middle of the Arch, and the number of the Courſes in the whole Arch will be odd, but if the laſt ſpace happen to fall juſt upon the middle Line

Line EF, as it doth in the Fig. then the joint is the middle of the Arch (but if it ſhould happen neither to come even to the Line, nor want an Inch of it, then you muſt open or ſhut the Compaſſes a ſmall matter, and begin again till it doth come right) and the number of the Courſes in the whole Arch is an even Number.

Note, When the Number of all the Courſes in the Arch is an even Number, then you muſt begin the two ſides contrary, *viz.* A Header to be the lower Brick of the firſt Courſe on one ſide (or half) of the Arch, and a ſtretcher the lower Brick of the firſt Courſe on the other ſide (or half) of the Arch: And contrariwiſe, if it happen that the Number of the Courſes be an odd Number, as 25 or 27, or ſuch like, then the firſt Courſes of each half of the Arch muſt be alike, that is, either both Headers or both Stretchers at the bottom.

Thus having deſcribed the Arch, the next thing to be done is to make the Sommering Mould, which to do, get a piece of thin Wainſcot (being ſtreight on one edge, and having one ſide Plained ſmooth to ſet the Bevil ſtrokes upon) about 14 Inches long, and any breadth above two Inches, then laying your Ruler, one end at the Centre P, and the other end even in the Skew-back Line, clap the ſtreight edge of the Wainſcot cloſe to the Rule, ſo that the lower end of the Wainſcot may lye a little below the Line AB, then take away the Centre Rule, but ſtir not the Wainſcot, and laying a Ruler upon the Wainſcot juſt over the Line CD, ſtrike a Line upon the Wainſcot, then ſet one Point of the Compaſſes being at the width of a Courſe (*viz.* 2 Inches) upon that Line, ſo that the other Point being turned about, may juſt

touch the ſtreight edge of the Wainſcot, (as you did before in dividing the Courſes) then make a Prick on the Line on the Wainſcot, and laying your Centre Rule upon it, and on the Centre P, draw a Line upon the Wainſcot by the Ruler with a Pencil, or the Point of a Compaſs, and cut the Wainſcot to that Line, and make it ſtreight by ſhooting it with a Plain, then your Wainſcot will fit exactly between any two Lines of the Arch; you may let it want the thickneſs of one of the Lines, or ſome ſmall matter more, which is enough for the thickneſs of a Mortar; The length of your Stretcher in this Arch may be 8 Inches and $\frac{1}{4}$, and the Header 3 Inches and $\frac{1}{4}$, but if your Arch be but 11 Inches in height, then make your Stretcher 7 Inches and an half long, and the Header 3 Inches $\frac{1}{2}$; One piece of Wainſcot will ſerve both for the length of the Stretcher, and the length of the Header, making it like a long Square or Oblong, whoſe ſides are 8 Inches $\frac{1}{4}$, and 3 Inches and $\frac{1}{4}$.

Then take a Bevil, and laying the inner edge of it ſtreight with the Line A B, and the Angle of the Bevil juſt over the Angle at A, take off the Angle that the Skew-back Line A C makes with the Line A B, and ſet it upon the ſmoothed ſide of your Sommering Mould for the Bevil ſtroke of your firſt Courſe; then drawing your Bevil towards E, ſtreight in the Line, until the Angle of the Bevil be juſt over the Angle that the ſecond Sommering Line makes with the Line A B; when it is ſo, draw the Tongue of the Bevil to lye even upon the ſecond Sommering Line, (in brief, cauſe the Bevil to lye exactly on the Line A B, and on the ſecond Sommering Line) then take up your Bevil and lay it on the Mould, and ſtrike that Bevil Line on the Mould with the Point of the Compaſſes about

bout half a quarter of an Inch distant from the first, and that is the Bevil of the underside of the second Course; proceed thus until you come to the middle Line E F, but after you have set 3 Bevil Lines upon your Sommering Mould, leave about $\frac{1}{4}$ of an Inch between the third and the fourth, and so likewise between the *6th* and the *7th*, and the *9th* and *10th*, which will be a great help to you in knowing the Number of each Line on the Mould.

The Moulds for the other half of the Arch, namely E B, are made after the same manner, but the Arch is described from a Centre beneath P, as Q, which causeth a less Skew-back (*viz.* B D.)

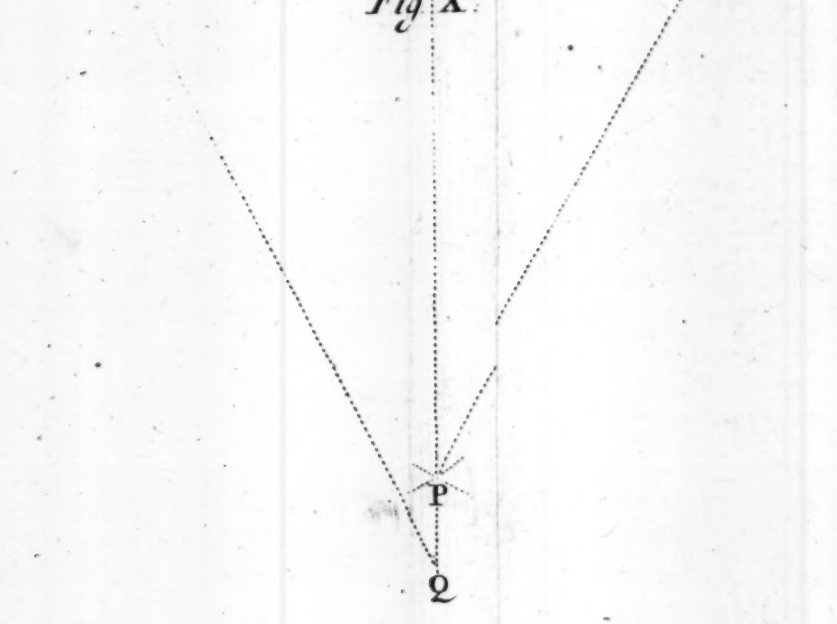
The diminishing of the Sommering Mould to any Skew-back may be found by the Rule of Three, by dividing a Foot into 10 equal parts, and each of those into 10 parts, so that the whole Foot may contain 100 parts, then proceed thus. The upper Line C F, will be 309, that is, 3 Feet and almost one Inch, and the lower Line A E will be 252, that is, 2 Feet and an half an $\frac{2}{100}$; and the upper part of the Sommering Mould will be 17 almost, that is, two Inches of such whereof there are 12 in a Foot; having these three Numbers (*viz.* 309, 252, 17) work according to the Rule of Three, and you will find 13 and $\frac{4}{7}$ of 100 parts, that is almost 14 (such parts whereof there are 100 in a Foot, Line measure) for the breadth of the lower part of the Mould.

You may likewise find it Geometrically thus.

Having drawn the upper Line and under Line of the Arch, as C F and A E, and drawn any Skew-back, as suppose A C in [*Fig.* X.] make at discretion the

Angle GCH in [*Fig.* XI.] then take the upper Line CF, and ſet it from C to F, alſo take the lower Line AE, and ſet it from C to E, and draw the Line EF, then take the thickneſs of your Brick, which ſuppoſe to be 2 Inches, and ſet it from F to G, and draw GH parallel to FE, I ſay FG is the breadth of the upper part of the Sommering Mould, and EH the breadth of the lower part. Then make your Sommering Mould true to thoſe two Lines, and beginning in the middle Line FE, deſcribe the ſtreight Lines by the Mould from the Key FE, until you come to the Skew-back AC, and then take off the Bevil Lines, and ſet them on your Sommering Mould; with which I conclude this Firſt Book of Geometry, being as it were an Introduction to that which follows.

Fig X



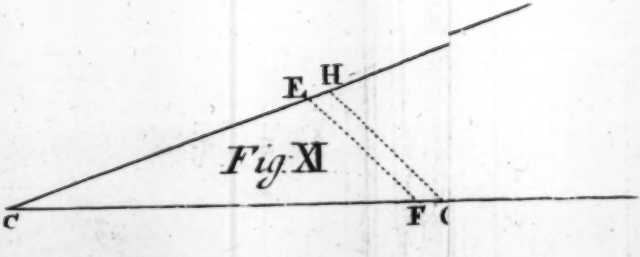


Fig XI

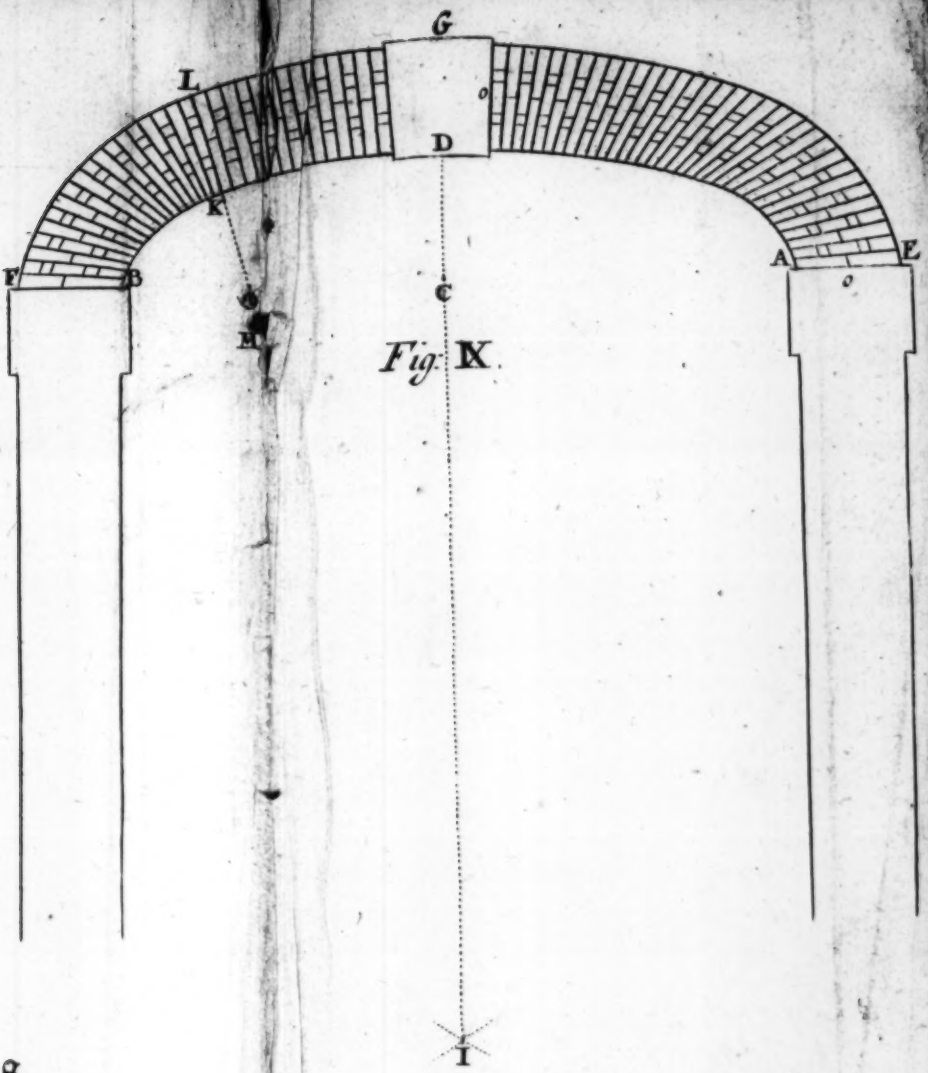


Fig. IX.

A GARDEN OF GEOMETRICAL ROSES; OR SOME PROPOSITIONS

Being hitherto Hid, are now made Known.

The Second Book.

Written in Latin

By *THOMAS HOBBES*,

And now Translated into English

By *VEN. MANDEY*.

LONDON,

Printed Anno MDCLXXXI.

Geometrical Roſes.

PROP. I.

Of cutting a right Line in extream and mean Proportion.

LEt there be deſcribed a Square A B C D, and let each ſide be divided in the middle in E, F, G, H; and F E, G H, being joyned, they will cut each other in the Centre of the Square at I. Likewiſe from the Centre D, let there be deſcribed a Quadrant D A C, cutting F E and G H in K and X.

Laſtly, Let E X be drawn; I ſay E X is equal to the greater Segment of the right Line E F, or of the ſide A B, being divided in extream and mean Proportion.

Let F X be drawn, and with that Semidiameter deſcribe an Arch of a Circle X z, cutting F E in z. Likewiſe with the Semidiameter E X, deſcribe an Arch of a Circle X y, cutting the ſame F E in y, and let the right Lines X z, X y be drawn.

Now the Angle E z X, is equal to thoſe two Angles z F X, F X z, becauſe theſe two Angles are within, the other is without the Triangle z F X.

Again

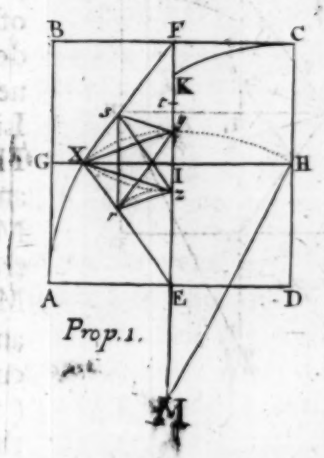
Again, the ſame Angle E z X for the ſame cauſe is equal to the two Angles X y z, y X z. And X z y and F X z are equal. Wherefore alſo the two Angles X F z, and z X y are equal to one another.

Therefore the two Triangles F X z, y X z have two Angles equal to two Angles of the Triangle z X y, and by conſequence three to three. Therefore they are Equiangled. Wherefore, as X F or E X, to X z; ſo is X z or X y, to z y. Therefore X z (or X y) is a mean Proportional between E X (or E y) & z y.

In the right Lines E X, F X, let there be taken E r, F s, and either of them equal to E z, or F y. Let r s, and s z be joyned. Then r X, X s will be equal. Alſo r s will be parallel to the right Line E F; and therefore the Altern Angles r s y, F y s, and likewiſe the Altern Angles s r y, E z r, are all equal to one another; and F s y r, and E z s r, will be Rhombus's both alike. Therefore the right Lines F s, s z, r y, r E, E z, X y, are equal to one another.

And becauſe it is manifeſt that X y is a mean Proportional between E y and z y; alſo F y, (equal to X y) will be a mean Proportional between E y and z y.

Therefore it will be, as E y, to F y, ſo F y, to z y; and beng Compounded, as E y more y F, (that is E F) to F y



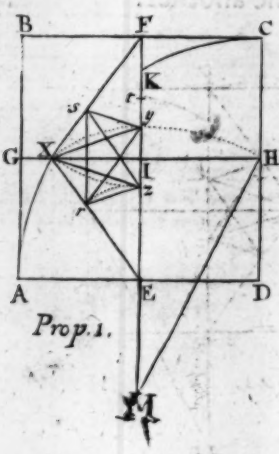
Prop. 1.

more

more yz (that is Fz or Ey) so likewise is Ey, to Ez. Therefore EF is divided in z and y, so that the whole Line EF, a part Ey, and the remaining part Ez; Likewise FE, a part Fz, and the remaining part Fy, are continual Proportionals. Therefore Ey, that is EX, is the greater Segment of the right Line EF, being divided in extream and mean Proportion, which was to be demonstrated. Let him reprehend it that can.

ANIMADVERS.

To him that rightly considers, it is a Paradox: Neither *Euclid* nor any other hath taught this. Who ever thought, that right Line which joyns half one side of the Square, with one third part of the Quadrant inscribed within the Square, to be the greater Segment of the side cut in extream and mean Proportion? Those Segments, of all hitherto, have been designed after this manner. Produce the right Line IE in M, so that IE, EM may be equal, and IM equal to the side. Moreover from the Centre M, with the Radius MH or MG, describe an Arch of a Circle, it will cut off a part from EF (suppose Ey) equal to the greater Segment. But that the right Line EX should



Prop. 1.

ſhould be equal, they never, nor any one did declare. Therefore, Reader, beware leſt thou truſt too much in an uncertain thing unadviſedly, for as much as we take things near the truth for true things. Examine the Demonſtration, and confer with true or ſurd Numbers, or conſult Algebraiſts.

CONSECT. I.

THe Lines sy, rz, being drawn about, will make the Figure of a Pentagon (*viz.* a five ſided Figure) Xsyzr, Equiangled and Equicrural. For the Angles Xsr, Xrs are equal, becauſe the right Lines Xs, Xr are equal; and either of thoſe Angles are equal to the Angle at F, becauſe sr, Fy are Parallels; and the Angles rsz, rsy are either of them equal to the ſame Angle at F, becauſe Es, Fr are Rombus's alike. Alſo the Angle yXz appears to be equal to the Angle at F. Likewiſe the Angle Xzr, is equal to the Angle yXz. becauſe rz, Xy are Parallels. Laſtly the Angles Xzs, Xyr, are equal to the ſame Angle yXz, becauſe the Baſes Xs, Xr, yz are equal.

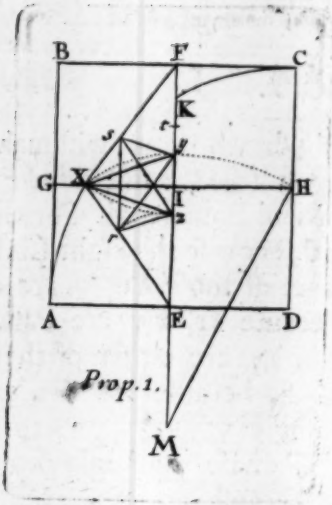
CONSECT. II.

THe Square of the greater Segment Ey, is equal to two Squares (to wit) to the Square of the right Sine of 30 Deg. and to half the Square of ſo many Deg. of the Line of Chords. For EI is equal to the right Sine of 30 Deg. And becauſe (the Chord XK being drawn) the Triangle XKI, is made Equicrural and right Angled, the Square of XI will be half the Square of the Chord XK; therefore

Ey,

Ey, that is, EX being Squared, contains as much as the Square of EI and XI added together.

CONSECT. III.



Ky is the half of FI: cut FI in the middle at t; therefore becauſe Fy more yI, and It more tF are equal; By how much Fy is greater than Ft, by ſo much tI is greater than yI; But Fy is greater than Ft, by ſo much as ty is.

Wherefore alſo tI, that is Ft, is greater than yI, by the ſame quantity ty. And Ft is divided in K, as tI in y. For becauſe Fy more yI, and IK more KF are equal; by how much Fy is greater than IK, ſo much is FK greater than yI.

Therefore when as Ft, is made equal to ty more yI; Ft will be divided in K, as tI in y.

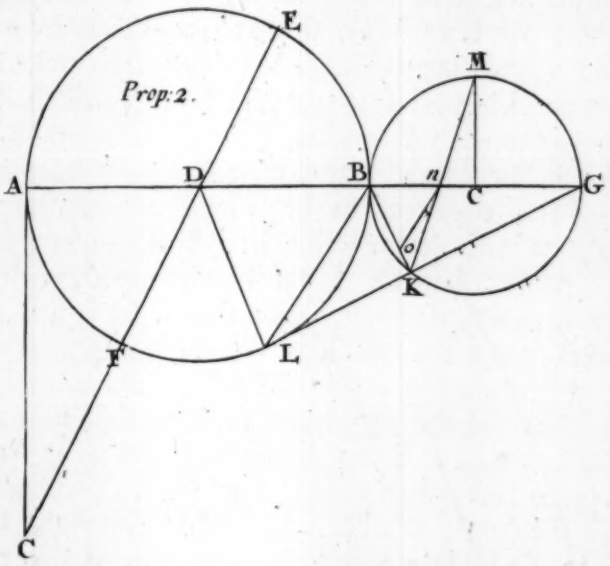
Wherefore tK is equal to yI; and Ky equal to FK more yI. Therefore Ky is equal to the half of FI.

PROP.

PROP. II.

One Segment being given of a right Line divided in extream and mean Proportion, to find the other.

LEt the right Line given be A B, being the greater Segment of any right Line. Cut A B in the middle in D, and from the Centre D, with the Interval (or Semidiameter) A D, describe the Circle A F B E. Then to the Point A, raise a Perpendicu-



lar

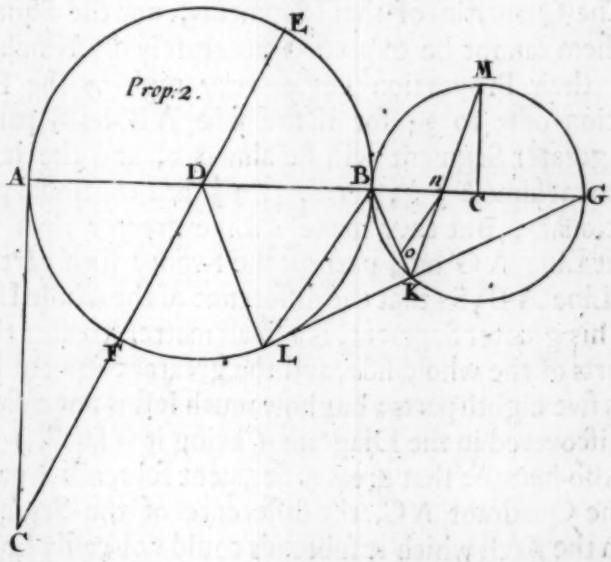
lar AC, equal to the given Line AB; and through the Centre D, draw a Line to the Concave Periphery CE, to which let AG be made equal.

I ſay the whole Line AG, hath the ſame Proportion to AB, that AB hath to BG. For, becauſe (by the 36 of the third Book of *Euclid*) as EC, is to EF; ſo is EF, to FC. Alſo as AG, together with his equal EC, is to AB and his equal EF, ſo will AB, be to BG, and his equal FC. Wherefore (by the definition of extream and mean Proportion) the right Line AG is divided in extream and mean Proportion in the Point B.

Again let BG be the leſſer Segment of any right Line. Cut BG the given Line in the middle in C. And from the Centre C, with the Semidiameter CB, deſcribe a Circle BMG. In which circumference, the Quadrant BM being taken, draw by the Point n the right Line MK, ſo that Bn be a third part of the given Line BG. Then draw BK and GK, and in the Line BK, take a part Bo, which ſhall be equal to the right Line Bn, draw the Line o—n, to which let there be a Parallel Line drawn, BL, cutting GK being produced in L. Finally, raiſe a Perpendicular to GL in the Point L, which will cut GB, being produced in D. From the Centre D, with the ſpace DB, deſcribe the Circle BLA. I ſay, GA is to AB, as AB to BG.

For becauſe the Angle BMK inſiſts in the circumference of the Quadrant BM, it will be half a right Angle, and being drawn from the right Angle BKG, it leaves half a right Angle nKG. And becauſe the Angle BKG is divided by the right Line nK in the middle, the right Line Kn, will cut the right Line BG, ſo that as Gn, is to nB; ſo will GK be

be to KB (by the 3 of the 6 of *Euclid.*) But Gn is by Conſtruction double to Bn. Therefore alſo GK is double to KB. And becauſe Bo is put equal to Bn, the Angle onB will be equal to the Angle Bon. And becauſe no and BL are parallels, the Angle LBD will be equal to the Angle onB. Likewiſe becauſe LD and KB are Parallels and Perpendiculars, the Angles DLB, and Bon will be equal. Likewiſe Bon and Bno are equal. Therefore DLB and Bno are equal. But Bno and DBL are equal, therefore DLB and DBL are equal. Therefore alſo the right Lines DL, DB being Chord Lines are equal. Likewiſe the Circle deſcribed with the ſpace (or Semidiameter) DB will paſs by the Point L. And becauſe DL is perpendicular to GL. GL toucheth the Circle BLA in L. Therefore it



E . will

will be (by the 36 of the 3 of *Euclid*) as GA to GL, ſo GL to GB. And AB is equal to the ſaid GL. (For whereas the right Line GK is double to the right Line BK; ſo alſo the right Line GL, will be double to the right Line LD.) Therefore in the ſame manner the right Line BA is double, and for that cauſe equal to the right Line GL. Therefore alſo it will be as GA, is to AB; ſo AB to BG, and by conſequence we have added the other Segment of the right Line given, *&c.* which was to be done.

ANIMADV.

I ſee not wherefore he hath invented this new way of cutting proportional Lines, unleſs perhaps from this he thought to find the greatneſs of the Circle, which he ſhould ſeek for, by comparing the power of the greater Segment, with the power of the Semidiameter, not before known.

The Quantities of theſe Segments, nor the Squares of them cannot be expreſſed accurately by Numbers. But their Proportion comes very nigh to the Proportion of 5 to 3; for if the ſide AB be 8 parts, the greater Segment will be almoſt 5, and the leſſer Segment almoſt $3\frac{1}{8}$; for 8, 5, $3\frac{1}{8}$ are continual proportionals; But they make a Line greater than the right Line AG by $\frac{1}{8}$ part of the twenty fifth part of the Line AB; So that the difference of the whole Line and his greater Segment, is a ſmall matter greater than $\frac{1}{8}$ parts of the whole ſide, and the greater Segment leſs than five eighth parts; but how much leſs is not eaſie to be diſcovered in the Diagram (being it is ſmall.)

Alſo becauſe that greater Segment ſubtends $\frac{2}{5}$ parts of the Quadrant AC, the difference of the Segment from the Arch which it ſubtends could not eaſily be diſtinguiſhed.

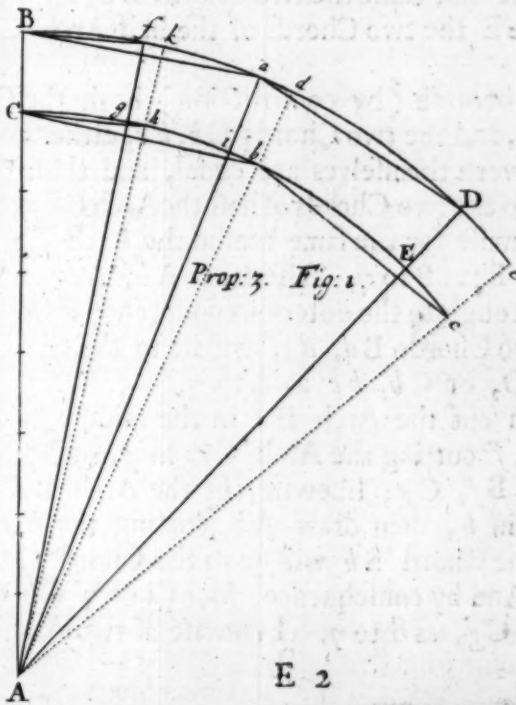
PROP.

PROP. III.

Of Regular Polygons.

In a Circle given to deſcribe a Regular Heptagon.

WHatſoever right Line, ſuppoſe A B, let it be cut in eight equal parts, whereof let A C be ſeven. Then from the Centre A, with the Semidiameters A B, A C, deſcribe two Circles. Moreover, take one eighth part of the Perimeter of the out-



ward Circle (to wit) B D (which is eaſily done) And draw the Line A D, cutting the Inner Circle in E, which will cut off the eighth part of it C E.

Divide the Arch B D in the middle in *a*, and draw the Chord Lines B *a*, *a* D; likewiſe in the Arch C E apply the right Line C *b* equal to B *a*, and again *b c* equal to the ſame B *a* or *a* D, for they are equal.

I ſay a right Line C *c* being drawn, is the ſide of a Heptagon in the Circle C E. For becauſe A B is to A C, as 8 to 7; ſo alſo the Perimeter of the Circle B D to the Perimeter of the Circle C E will be as 8 to 7.

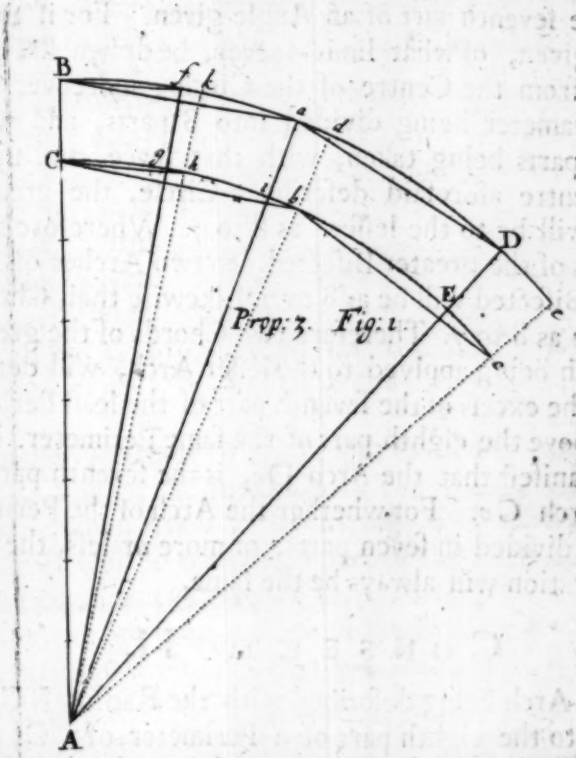
Alſo becauſe the Sectors A B D, and A C E are alike, and the Triangles A B D, A C E are alike, both the Arch B D to the Arch C E, and the Chord B D to the Chord C E will be as 8 to 7.

For the ſame cauſe the two Chords B *a*, *a* D, will be to C *i*, *i* E the two Chords of the half Arch C E, as 8 to 7.

Now becauſe (by conſtruction) both the Chords B *a*, *a* D, and the two Chords C *b*, *b c*, either to other and between themſelves are equal, and thoſe Chords will be to the two Chords of half the Arch C E, as 8 to 7. And likewiſe for the ſame reaſon the Arch C *c* to the Arch C E as 8 to 7. Alſo take A *b*, A *c*, and draw them through to the outer circumference in *d* and *e*, thoſe two Chords B *d*, *d e*, will be to the two Chords B *a*, *a* D, or C *b*, *b c* as 8 to 7.

Again cut the Arch B *a* in the middle in *f*, and draw A *f* cutting the Arch C E in *g*, and draw the Chords B *f*, C *g*; likewiſe cut the Arch B *d* in the middle in *k*, then draw A *k* cutting the Arch C E in *h*, the Chord B *k* will be to the Chord C *h*, as 8 to 7. And by conſequence, four Chords C *h* to four Chords C *g*, as 8 to 7. Likewiſe if two Arches C *h* and

and C*g* be cut in the middle, and their Biſegments cut *ad infinitum*, every Chord of thoſe, to every Chord of theſe, will be in proportion, as 8 to 7. But the Chords thus taken infinite in any Arch are (all together) the Chord of the ſame Arch. Wherefore the Arch C*c* is to the Arch CE, as 8 to 7. And the Arch C*c* is the ſeventh part of the whole Perimeter by C. Therefore a right Line being drawn C*c*, is



the ſide of a Heptagon in the Circle by C. Likewiſe the right Line A*e* being drawn, will cut the ſeventh part of the Perimeter by B.

For if every one of the Chords to every one of the Arches which they ſubtend are not equal, thoſe ſeveral Arches and their Biſegments may be again Biſected, which is, contrary to the Suppoſition. Therefore the ſide of a Heptagon is found, which was propoſed.

Consect. I.

From this Demonſtration appears a method of finding the ſeventh part of an Angle given. For if to an Arch given, of what limits ſoever, be drawn ſtreight Lines from the Centre of the Circle; moreover the Semidiameter being divided into 8 parts, and 7 of thoſe parts being taken, with that ſpace, and upon the Centre aforeſaid deſcribe a Circle, the greater Arch will be to the leſſer, as 8 to 7. Wherefore two Arches of the greater Biſected, to two Arches of the leſſer Biſected will be as 8 to 7. Likewiſe their Chords will be as 8 to 7. Therefore two Chords of the greater Arch being applyed to the leſſer Arch, will determine the exceſs of the ſeventh part of the leaſt Perimeter, above the eighth part of the ſame Perimeter. For it is manifeſt that the Arch D*e*, is the ſeventh part of the Arch C*c*. For whether the Arch of the Perimeter be divided in ſeven parts, or more or leſs, the demonſtration will always be the ſame.

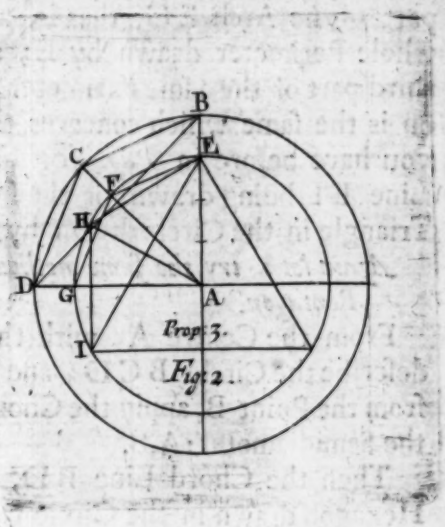
Consect. II.

An Arch being deſcribed with the Radius B C, is equal to the eighth part of a Perimeter of a Circle, whoſe Radius is A B. And to the ſeventh part of the Perimeter of a Circle, whoſe Radius is A C. And to a ſixth part of the Perimeter of a Circle, whoſe Radius is $\frac{4}{5}$ parts of the right Line A B, *&c.*

Let

Let us experience this our Method in known Polygons, and see whether or no by this, the side of an Equilateral Triangle may be found from a Tetragon, (*viz.* a Square.)

From the Centre A, describe the Circle BCD, whose Quadrant is B D. Then draw the Chord Line B D, which is the side of a Square inscribed within that Circle.



From the same Centre A, with the Semidiameter A E, (which let be to A B, as 3 to 4,) describe the Circle E F G; then the Circle by B, will be to the Circle by E, as 4 to 3.

Let the Quadrant B D, be cut in two in the middle in C, and let the equal Chord Lines B C, C D be drawn. Then draw A C cutting E G in F, the Arch E F will be the eighth part of the Perimeter by E, and equal to the Arch F G.

Therefore both the Arch and the Chords B C, C D will be to the Arch, and to the Chords E F, F G, as 4 to 3.

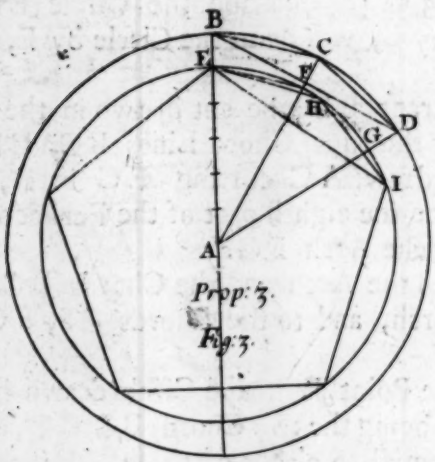
Then from the Point E in the Circle drawn by E, apply E H, H I, being the two Chords B C, C D, either of these being equal to either of those. Wherefore the two Chords E H, H I, are to the two Chords E F,

FG as 4 to 3. That is, as the Arch BD to the Arch EG. Wherefore as the Arch EI to the Arch EG, ſo is 4 to 3. Therefore the Arch GI, is the third part of the Arch EG, that is, a twelfth part of the whole Perimeter drawn by E; and the Arch EI, a third part of the ſame Perimeter. The Demonſtration is the ſame which concerns the Heptagon, which you have before in *Pag.* 50. Therefore the Chord Line EI being drawn, is the ſide of an Equilateral Triangle in the Circle drawn by E.

Again let us try the ſame method from a Hexagon to a Pentagon.

From the Centre A, with the Semidiameter AB deſcribe the Circle BCD; and in that circumference from the Point B, apply the Chord Line BD equal to the Semidiameter AB.

Then the Chord Line BD, will be the ſide of a Hexagon drawn in the Circle by B.



Prop. 3.
Fig. 3.

In the Semidiameter AB, take AE, which muſt be to AB, as 5 to 6. And from the ſame Centre A, with the Radius AE deſcribe the circle EFG Then the Perimeter by

by B to the Perimeter by E, will be as 6 to 5. Let BD be cut by a right Line AC (cutting the Circle drawn by E, in F) in the middle in C; and let the Chords BC, CD be drawn, equal to which apply the Chords EH, HI, in the Circle drawn by E. Then are the two Chords EH, HI, to the two Chords EF, FG, as the Arch BD to the Arch EG, that is as 6 to 5; and the Arch EI, to the Arch EG, as 6 to 5; and the Arch GI is a fifth part of the Arch EG, that is, the thirtieth part of the Perimeter; and the whole Arch EI, ſix of thoſe thirty parts (that is, a fifth part) of the whole Perimeter drawn by E. The Demonſtration is the ſame as in the Heptagon. Therefore the Chord Line EI, is the ſide of a Pentagon drawn in the Circle by E.

Cor. 1. Therefore the ſide of a Pentagon may be found without the work of cutting the Semidiameter in extream and mean Proportion.

Cor. 2. As from the outward Circle to the inward, the Demonſtration hath proceeded hitherto; ſo likewiſe it may proceed from the inward Circle to the outward. As from a Triangle given, may be found a Quadrat (or Square) and a Pentagon from a Square, and a Hexagon from a Pentagon, and ſo of the reſt.

For to the ſide of a Pentagon given EI, the two Chords EH, HI are given. Wherefore if to the Semidiameter AE, be added a fifth part of the ſame Semidiameter (to wit) EB, and the Arch BD be deſcribed; the two Chords EH, HI, will be equal to thoſe two Chords BC, CD, and equal each to other, and the Chord BD, the ſide of a Hexagon inſcribed in the Circle by B.

PROP.

PROP. IV.

Of the Proportion of crooked Lines, to crooked Lines in the circumferences of Circles.

1. AS a Circle is described from a Semidiameter with one Foot of a Pair of Compasses being fixed, and the other Foot carried about; so also it may be understood to be described from a right Line given, being bent uniformly, that is, so as the Angles be always equal; from which certain flexion or bending, if the Angles be conceived to be infinite in number, is described a Circle. For a Circle in its nature, differs nothing from a Polygon of an infinite number of sides.

And bending is a departing from streightness according to some Angle, which is crookedness.

2. And the crookedness of some to other some is greater or lesser; and therefore crookedness is quantity, and belongs to the Subject of Geometricians, and chiefly to those who write concerning the magnitude of a Circle, and the Inscription of Polygons in a Circle, although concerning this thing, nothing hath been delivered to us from the Ancients.

3. What Proportion an Angle in a circumference, to an Angle, in a circumference in the same Circle hath, the same Proportion hath the crookedness of the greater Arch to the crookedness of the lesser Arch. For since crookedness is no other thing than a bowing (by an Angle in a circumference) from streightness; It must be that the greater Angle makes the greater crookedness. From whence it follows, that the crookedness of Arches in the same Circle, be to each other like their Angles.

4. In

4. In divers Circles the crookedneſs of the greater Perimeter, is leſs than the crookedneſs of the leſſer Perimeter, in the Proportion of the Radius to the Radius, or of the Diameter to the Diameter reciprocal. For in great Circles, as in the great Circle of the Earth no crookedneſs can be diſcerned in a long ſpace, but in a Ring it is every where diſcerned; therefore in indifferent Circles, the leſſer Circles have the greater crookedneſs, for this very cauſe, becauſe the Diameter is leſſer; which is manifeſt by the light of nature.

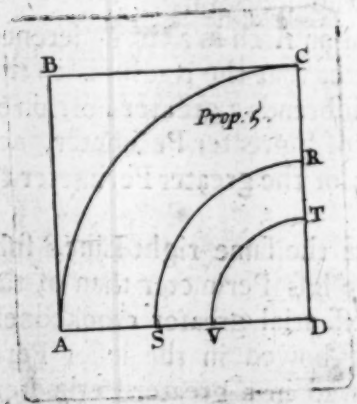
5. (If the Proportion of an Arch in a circumference be the ſame to the Perimeter that the Radius is to the Radius) Any ſame Chord ſubtends a greater Portion of his own Perimeter, than of a greater Perimeter, according to the Proportion of the greater Perimeter to the leſſer.

For the cauſe wherefore the ſame right Line, ſubtends a greater part of the leſs Perimeter than of the greater, is the only and eſſential greater crookedneſs of the ſame Line, being bowed in the leſſer Perimeter than when it is bowed in a greater Perimeter. Therefore if the crookedneſs of a leſſer Arch to the crookedneſs of a greater Arch in a Semicircle be as 8 to 7, the Chord which ſubtends the eighth part of the greater Arch, will ſubtend the ſeventh part of the Semi-Perimeter of the leſſer (I ſay in a Semi-circle, becauſe the crookedneſs of a Perimeter beyond a Semicircle proceeds a contrary way, and becauſe the Subtenſes of the Arches, which together with themſelves in a Semi-circle encreaſe; but beyond a Semicircle they all decreaſe.)

6. Alſo therefore an Angle in the circumference of a leſſer Perimeter (becauſe the Subtenſes are equal) will be to an Angle in a greater Perimeter, as the greater Perimeter to the leſſer. PROP.

PROP. V.

A mean Proportional between the Semidiameter of a Circle, and $\frac{2}{5}$ parts of the same is equal to $\frac{2}{5}$ parts of the fourth part of the Circle.



DEscribe a Quadrant of a Circle DAC, and compleat the Quadrat ABCD; in the side DC, take DT being two fifth parts of the side DC, and between DC and DT, let there be taken a mean Proportional DR, so that DC, DR, DT, may be continual Proportionals; also describe the Quadrantal Arches RS, TV; Then the Arch TV is two fifth parts of the Arch CA.

I say the Arch TV, and the right Line DR are equal.

Let it be supposed that there is a right Line given equal to the Arch AC, and from it describe a Quadrantal Arch (*viz.* with the right Line given being Radius, describe the fourth part of a Periphery) the Line DC, CA, and the Quadrantal Arch above CA are continual Proportionals.

Let there be writ a part DC, CA, the Arch above CA ∴
And under these DR, RS, the Arch above RS ∴
Again under these DT, TV, the Arch above TV ∴

Then

Then the Antecedents to the Conſequents will be through all the Orders or Ranks forward, as the Semidiameter to the Arch of a Quadrant, and backward, as an Arch of a Quadrant to the Semidiameter.

Therefore as DC to RS, ſo is RS to the Arch above TV; therefore DC, RS, and the Arch above TV will be continual Proportionals. And becauſe DC, CA, and the Arch above CA are likewiſe continual Proportionals, and have the firſt Antecedent DC common; the Proportion of the Arch above CA to the Arch above TV, will be (by the 28 of the 14 of *Euclid*) in Duplicate Proportion of CA to RS; and for the ſame cauſe the Arch above RS, is a mean Proportional between the Arch above CA, and the Arch above TV. Now if DC be greater than RS, likewiſe RS will be greater than the Arch above TV; and the Arch CA greater than the Arch above RS. Wherefore, when as DC, CA, and the Arch above CA are continual Proportionals, the Arch above TV, and the Arch above RS, and the Arch above CA cannot be continual Proportionals, becauſe it is Demonſtrated to the contrary. Therefore DC is not greater than RS.

Again let RS be ſuppoſed to be greater than DC; then the Arch above RS will be a mean Proportional between the Arch above TV, and the greater Arch above CA. Therefore the inconvenience will return.

Wherefore the Semidiameter DC is equal to the Arch RS; and by conſequence the Arch TV, that is two fifths of the Arch CA, and the right Line DR are equal, that is, a mean Proportional between the Semidiameter and two fifth parts of the ſame, is equal to two fifth parts of the fourth part of the circumference of the Circle. Which was to be Demonſtrated.

PROP.

PROP. VI.

LEt there be described a Quadrate ABCD, and let it be cut in the middle both ways by EF, GH, also let it be cut by the Diagonals AC, BD (concurring in the Centre I) four ways.

Moreover, between DC and two fifth parts thereof, let there be taken a mean Proportional DR, and join AR, cutting EF in *a*, let it be produced until it meet with the side BC being produced in *b*.

I say that the right Line B*b* is Quintuple to (*viz.* five times as long as) the right Line E*a*, or the fifth part of the Arch AC (or it may be thus rendred the right Line B*b* is equal to the Quadrantal Arch AC.)

Because the Triangles ADR, AB*b* are alike, and the Arch of a Quadrant described from DR is equal to the side DC, or AB, likewise the Arch described from AB will be equal to the right Line B*b*. Therefore whereas DR is two fifth parts of the Arch AC and by consequence B*b* five of those fifths; B*b* will be five times as long as the right Line E*a*, which is the half of the right Line DR, and the fifth part of the Arch AC, or the right Line B*b*. Which was to be Demonstrated.

Therefore if in the side BC, be taken a part Bi equal to E*a*, that part B*i* being five times repeated will end in *b*.

A Compendious Exposition.

If two right Lines whatsoever have a mean Proportional between them, which is equal to the side AB it will be as the Arch AC to one of them, so reciprocal

procally the other of them will be to two fifth parts of the ſame Arch AC: as in example, becauſe the ſide AB is a mean Proportional between the Arch AC, and two fifths of the ſame; and the ſame a mean Proportional between the Diagonal Line AC, and the half of it DI. The right Angle under the Arch AC, and two fifth parts of the ſame, will be equal to the right Angle under BD, and the half of it DI. Therefore as the Arch AC is to his Diagonal AC, or BD, ſo will the half Diagonal DI be to DR. And as the half of the Arch AC, to the half of the Diagonal, ſo is the half Diagonal to DR.

CONSECT.

From hence appears the magnitude of the eighth part of the whole Perimeter. For if to the right Line DR, be added R*z*, being a fourth part of DR, the whole Line D*z*, will be five of thoſe ten parts, that is half the Arch AC. For when as DR is two fifths, that is, four tenths, D*z* will be five tenths.

PROP. VII.

THe ſame Lines continuing, draw the right Line DF. I ſay DF is a mean Proportional between the whole Arch AC and his half.

Draw the right Line R*r* parallel to the ſide BC, cutting the Diagonal DB in *r*, and DF in *d*. Then becauſe DF cuts the ſide BC in the middle in F, the right Line D*d* will cut the right Line R*r* in the middle in *d*. Let LN be drawn Parallel to the ſame ſide BC, cutting DF in *e*, and the ſide DC in N. Then DN and NL will be equal, and either of them equal to the Semidiagonal DI, and NL will be divided in the middle in *e*. There-

Therefore becauſe by the Compendious Expoſition of the precedent Prop. D*z*, DN, DR, are continual Proportionals; If with the Radius D*z* be deſcribed an Arch *z f*, it will cut the right Line NL in *e*; likewiſe if with the Radius DN be deſcribed an Arch of a Circle NI it will cut the right Line R*r* in *d*.

Then deſcribe the Quadrantal Arch NIO, which (as it is ſhewn) will paſs through *d*. Then becauſe DR is the Radius of a Circle, whoſe fourth part of the Perimeter is equal to the ſide DC; the right Line D*d* or DN will be the Radius of a Circle, whoſe fourth part of the Perimeter is equal to the right Line DF. But the right Line DN is the Radius of a Circle, whoſe fourth part of the Perimeter is the Arch it ſelf NIO.

Therefore the right Line DF, and the Arch NIO are equal. And the Arch NIO is a mean Proportional between the Arch AC and his half, therefore alſo the right Line DF is a mean Proportional, *&c.* which was to be Demonſtrated.

Cor. If from a Point *z* be drawn a right Line *z c* parallel to the ſide BC, cutting the Diagonal DB in *c*; D*c* will be equal to DF. For DC will be a mean Proportional between the double and ſimple of D*z*. Likewiſe it is made appear that D*z* is equal to half the Arch AC.

CONSECT. I.

The Arch AC, that is the right Line B*b*, is a mean Proportional between the ſide DC, and the Quintuple of (*viz.* a Line five times as long as) the half ſide BF. For ſeeing that two fifths of the ſide DC, the right Line DR, and five of thoſe fifths of the ſide

ſide DC are continual Proportionals; If the Proportion be continued, DR, DC, and the mean Proportional between DC, and the Quintuple of the half ſide BF will be continual Proportionals. Therefore a Quadrate being made or drawn from the right Line B*b*, is equal to ten Quadrates drawn from the half ſide BF. From whence alſo it follows that a Quadrantal Arch being deſcribed from the Arch AC, being extended in a right Line, is Quintuple to the half ſide BF.

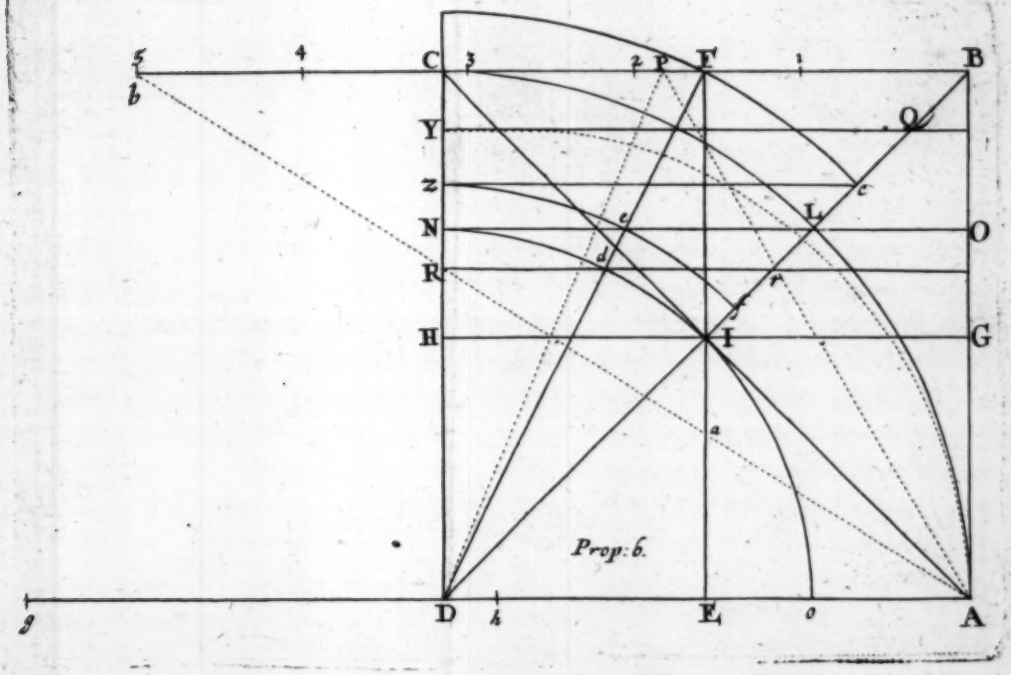
CONSECT. II.

The ſame Lines remaining, to the ſide AD let there be added in a direct Line D*g*, equal to the right Line D*z*, that is equal to the Arch CL; moreover cut the whole Line A*g* in the middle in *h*, then from the Centre *h*, with the Radius *h*A, or *h g*, deſcribe the Arch of a Circle AY, cutting the ſide DC in Y. Then DY will be a mean Proportional between the ſide DC and D*z*. And a right Line YQ being drawn Parallel to the ſide BC, cutting the Diagonal DB in Q, YQ will be the ſide of a Quadrant (from *Archimedes* Demonſtration) equal to the Secto DCL, or an eighth part of the whole Circle deſcribed from the Semidiameter DA.

CONSECT. III.

From hence it follows, that the Arch AC *is equal to the Compound of the ſide* BC *and a Tangent of* 30 *Degrees.*

For becauſe DC is a mean Proportional between the Arch AC and DR; if to DR, and DC there be taken a third Proportional, it will be equal to the ſide BC, together with the Tangent of 30 Deg.


5
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But the Diagram muſt be renewed (or drawn anew) Therefore let ABCD be a Quadrate, and let it be divided into four equal parts by the right Lines EF, GH; alſo let it be divided into four equal parts by the Diagonal Lines AC, BD, concurring in the Centre I. Alſo draw the Quadrantal Arches AC, BD cutting EF and GH, in K and X.

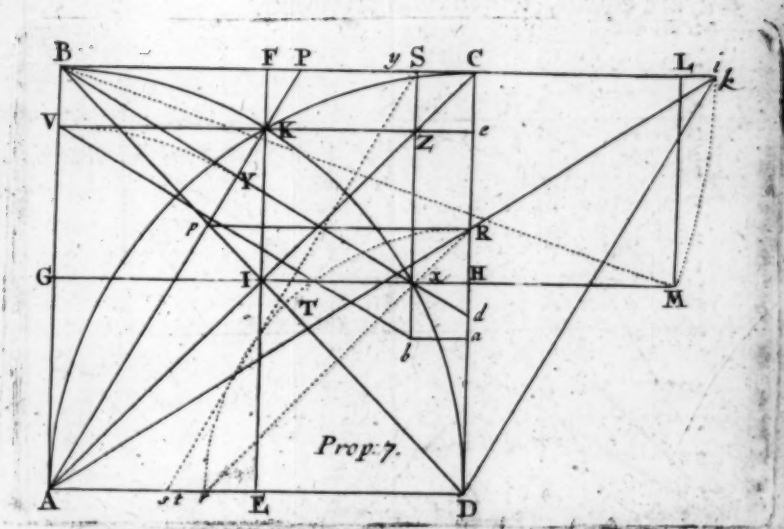
Produce the Lines BC, GH, and let there be taken the right Lines CL, HM, either of them being equal to half the ſide, and let BM be joined. Then BM being taken as a Semidiameter, with it, from the Centre B, deſcribe an Arch of a Circle, cutting BC, being produced in *i*. Then becauſe BL being ſquared, contains as much as nine Quadrates (or Squares) made from the half ſide, and LMHC one of them quadrates. Alſo the right Line BM, that is, B*i* being ſquared (to wit, a quadrate being drawn with four Lines, each of them being the length of B*i*) that Quadrate contains as much as ten Quadrates made from the half ſide CH, and therefore B*i* is equal to the Arch AC, or BD.

Draw AK and produce it to the ſide BC in P; AP will be a Secant of 30 Deg. and BP a Tangent of 30 Deg. and AP the double of BP. To the ſide BC add C*k* equal to BP, and the whole Line B*k* will be compoſed of the ſide BC and a Tangent of 30 Deg. BP or C*i*.

It remains therefore to be Demonſtrated, that *i* and *k* meet in the ſame Point.

Let A*i* be joyned cutting DC in R; and with the Radius DR, deſcribe a Quadrantal Arch R*r*, cutting the Diagonal Line BD in T.

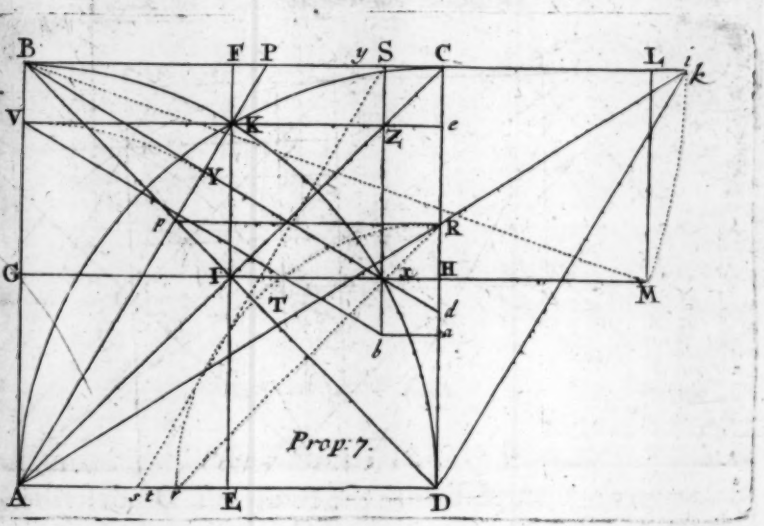
From



From the Centre T with the Radius TD, deſcribe an Arch of a Circle cutting BC in S, and produce ST to the ſide AD in S; then are DR, DT, TS equals.

Then becauſe AB is a mean Proportional both between AP and EK, and between B*i* and DR, likewiſe between the Diagonal BD and its half CI; it will be as B*i* (or the Arch AC) to BD, ſo CI to DR or ST. And likewiſe as BD to AP, ſo reciprocally EK to DR or ST. Wherefore if there be taken in the ſide BC, a certain right Line equal to EK, ſuppoſe the right Line B*y*, and from thence to the ſide AD be drawn *yt* parallel to AP, it will be as B*i* to *yt*, ſo B*y* to ST. Wherefore the right Line *yt* will paſs by T, and *y*T will be equal to ST, which is abſurd. Then BS is equal to EK, and S*s* equal to AP. Therefore becauſe AP is equal to DK, DK will be parallel to S*s*. Then

as B*k* to D*k*, ſo is BS to ST. Therefore B*i* and B*k* are equals, which was propoſed.



CONSECT. IV.

From hence it follows, that the right Line HR (which is the difference whereby two fifths of the Arch AC exceeds the half ſide DH) is equal to FK the verſed Line of 30 Degrees.

For if in the ſide AB, there be taken AV equal to EK, and with the Radius AV be deſcribed an Arch of a Circle, cutting AP in Y, the Arch VY will be equal to the half ſide BF or BY, and AY equal to AV; and therefore BY will paſs to *x*, and the ſame BY will be a Tangent of 30 Degrees in the Circle, whoſe Semidiameter is AV or EK. Then produce VK to the Diagonal AC in *z*, V*z* will be equal to EK. Wherefore a Perpendicular being dropt down from the Point S, it will paſs through *z* and

and x; and Bx being produced to the ſide DC in di Cd will be equal to Ci. But Sx is equal to hal the ſide, and therefore $z\,x$ equal to Kz.

In the ſide CD, let there be taken the right Line Ca equal to DR, and let $a\,b$ be drawn parallel to the ſide BC, cutting $z\,x$ produced in b, and let Vb be joyned. Then becauſe as Bi is (that is BC more the Tangent Ci) to the Secant Bd, ſo is BS to DR, that is to Sb; and as BC to the Tangent Cd, ſo Vz to Sx; if in Sx produced, be taken $z\,b$ equal to Sx, $z\,b$ will be a Tangent of 30 Degrees in the Arch deſcribed from Vz.

Then joyn Vb, it will be equal to the ſide BC, and becauſe $z\,b$ is a Tangent of 30 Degrees in the Circle, whoſe Radius is Vz, and Cd a Tangent of 30 Degrees in the Circle, whoſe Radius is AB; Bd and Vb will be parallels; and joyn Vb equal to the ſide BC, and BC more Cd will be equal to the right Line Bi.

Then becauſe it is as BC more Cd (that is Bi) to Bd, ſo Vz to DR; and as the ſame Bi to Vb, ſo Vb to DR, and Sb will be equal to DR. For theſe two Analogies can no way be conſtituted in any other Point of the right Line Sx.

Then the equals DH being taken from DR, and $z\,b$ from Sb, there remains HR, Sz, both equals; but Sz is equal to FK. Wherefore HR, FK, are equals.

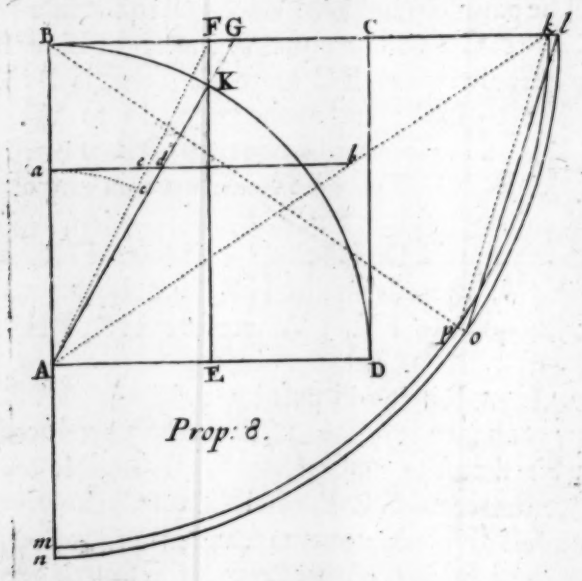
Cor. Joyn Rr it will paſs by x.

Alſo from hence it follows (Vz being produced to DC in e) that the right Line Re is double to the difference between GB the half ſide, and the greater Segment AB divided in extream and mean proportion. For (by the firſt Cor. of this Prop.) the fourth part of the ſide AB or DC, is equal to the right Line FK,

together with the difference between the half side, and the greater Segment of the side. Wherefore half the side, that is, *a e*, is equal to the double of F K, and to the double of the difference between the greater Segment of the side and the half side. But *a* R is double to F K; wherefore R *e* is double to the difference between the greater Segment of the side and the half side.

PROP. VIII.

AGain describe a Quadrate A B C D, also describe a Quadrantal Arch B D; and to the side B C, let there be added C *k* equal to a Tangent of 30 Degrees, and let the right Line A *k* be drawn, moreover in the side A B, let there be taken A *a* equal to



Prop: 8.

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the right Line A *c*, being made equal to the greater Segment of the ſide A B (divided in extream and mean Proportion). Then draw *a b* parallel to the ſide BC, cutting A *k* in *b*.

I ſay the right Line *a b*, is the ſubtenſe of two fifth parts of the Quadrantal Arch *k m*, deſcribed with the Semidiameter B *k*, and thoſe two fifths equal to the ſide A B.

For becauſe A *a* is the greater Segment of the ſide B C, being divided in extream and mean Proportion, it is alſo the ſide of a Decagon in a Circle, whoſe Semidiameter is A B (by the 4 Prop. of the 14. Elem. of *Euclid*) and ſubtends the tenth part of the whole Perimeter, that is, a fifth part of the half Perimeter, that is, two fifth parts of the Arch B D,

Therefore ſince it hath been ſhewn, that the right Line B *k* is equal to the Arch B D, it will be, as AB to A *a*, ſo B *k* (that is, the Arch B D) to *a b*.

Wherefore *a b* is the greater Segment of the right Line B *k* divided in extream and mean Proportion.

Apply to the Arch *k m*, the right Line *k p*, equal to *a b*; Then the Arch *k p* will be two fifths of the Quadrantal Arch *k m*, which is the firſt. But the Arch *k m* (by the 4 Corol. of Prop. 7) is equal to five half ſides of a Quadrate from A B. Wherefore the Arch *k p* is equal to the ſide A B, which remained to be Demonſtrated.

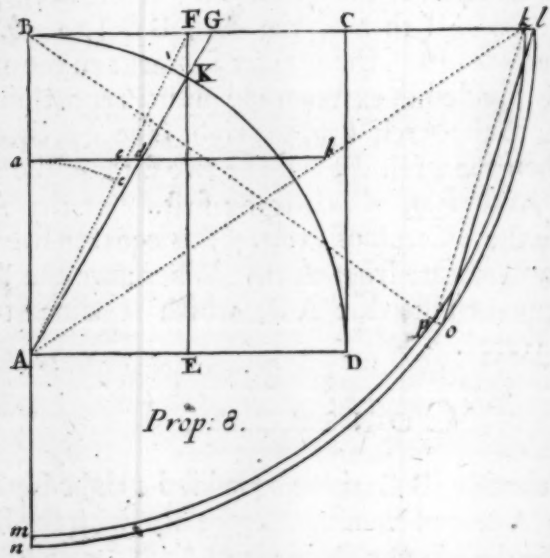
CONSECT. I.

If to the ſide B C there be added a right Line C *l* equal to A *a*, and from the Centre B, with the Interval B *l* be deſcribed a Quadrantal Arch *l n*, alſo joyn B *p* being produced to *l n*, in *o*: The Chord Line *l o*

being drawn, it will be equal to the Arch *k p*, or the ſide A B. For (by the 5 Prop. of the 13 Elem.) the ſide B C will be the greater Segment of the whole B *l*. Wherefore the ſide B C ſubtends two fifths of the whole Arch *l n*. Therefore ſince the Angle *k* B *p* is two fifths of the Angle *k* B *m*, likewiſe *l o* will be two fifths of the Arch *l n*. Wherefore the right Line *l o* which ſubtends the Arch *l o*, is equal to the ſide B C, and the ſame equal to the Arch *k p*.

Consect. II.

From hence it is manifeſt that both the Quadrate it ſelf A B C D, and all the right Lines from the Centre A to the ſide B C (if the work be produced) are cut from *a b* (if the work be produced) in ex-



Prop. 8.

tream

tream and mean Proportion. For the Quadrate ABCD is divided in the ſame Proportion whereby the ſide AB is in *a*; likewiſe from the ſame *ab* are divided all the right Lines drawn from A to BC (when the work is produced) in the ſame Proportion whereby AB was divided in *a*. Wherefore (by the 2 Prop. of the 14 Elem.) they are divided in extream and mean Proportion.

Cor. From hence a manifeſt and brief method appears of finding Proportional Segments of right Lines of what kind ſoever. As for example, if the greater Segment of a Secant of 30 Degrees be ſought. Draw the Line AK, and let it be produced to the ſide BC in G, which is the Secant of 30 Degrees; it will cut the right Line *ab* in *d*, and A*d* is the greater Segment, and the reſidue is the leſſer Segment. Or elſe the Section of a Tangent of 30 Degrees being ſought, which as aforeſaid is half the Secant, and the ſide of a Cube inſcribed in a Circle, whoſe Diameter is AB.

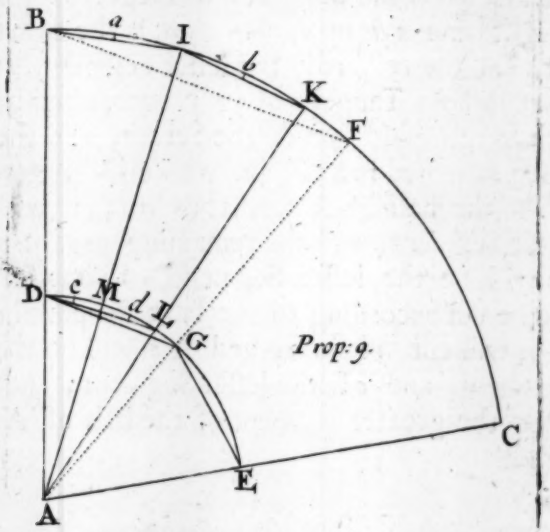
Take the half of A*d*, that is *ad*, it will be the greater Segment, and the remaining part of the Tangent will be the leſſer Segment. Likewiſe, if AF be to be cut according to the ſame Proportion, draw AF it will cut *ab* in *e*, and A*e* will be the greater Segment, and *e*F the leſſer Segment; and the half of A*e* the greater Segment of the half of AF.

PROP.

PROP. IX.

To cut an Angle given, into any Proportion given.

LEt the Angle given be BAC, and let the Proportion given be as AB to AD. Let the Arch ADE be described, and cut the Arch BC in the middle in F. Wherefore AF being drawn, it will also cut DE in the middle (suppose) in G. Then the Chords BF, DG being drawn, they will be between themselves as the right Lines AB, AD.



Apply the two Chords DG, GE, to the Arch BF in I and K, so that the Chords BI, IK may be equals to the Chords DG, GE, and either to other; also draw the Lines AI, AK, of which AK cuts the Arch DE in L, and AI cuts the same Arch in M.

I

I ſay the whole Arch BC being thus divided in K, That the Arch BC (or the Angle given BAC) is to the Arch BK (or to the Angle BAK) as the right Line AB to AD.

For the Chord BK is to the Chord DL, as AB to AD. Alſo as the two Chords BI, IK, to the two Chords BM, ML, ſo is AB to AD. But the two Chords BI, IK are by conſtruction equal to the two Chords DG, GE. Wherfore the two Chords DG, GE are to the two Chords DM, DL, as AB to AD. But as two Chords DG, GE, to two Chords DM, DL, ſo is the whole Arch DE, to the Arch DL; Which I thus make appear.

If the Arches BI and IK be cut in the middle in *a*, and *b*, likewiſe the Arches DM, ML being cut in the middle in *c* and *d*; and the Chords of the Biſegments in the Arch BK being drawn; likewiſe the Chords of the Biſegments in the Arch DL; alſo theſe will be as AB to AD, and they will always be ſo, if the Biſegments of the Biſegments proceed infinitely. Alſo the ſame is true in the Biſections of the Arches KC and LE. But the Chords of the Biſegments infinite in number are equal to the Arch it ſelf. For if all the Chords were leſs than all their Arches, there might yet a Biſection proceed; which is contrary to the ſuppoſition.

Therefore the Arch DE is to the Arch DL, as the Arch BC to the Arch BK, as the given Line AB, to the given Line AD, and ſo alſo the given Angle BAC to BAK. Therefore the Angle given BAC is cut in K, in the Proportion given of AB to AD. Which was to be done.

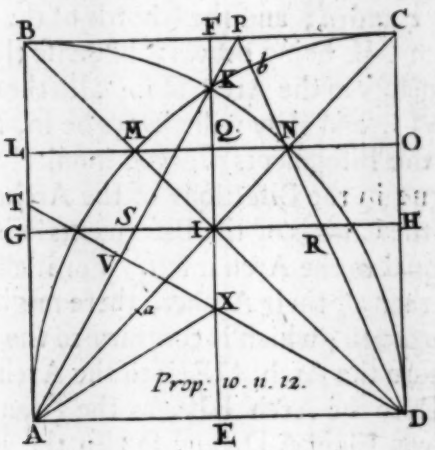
PROP.

PROP. X.

Of Sines, Subtenſes, and other Lines in the Quadrant of a Circle.

THE Tangent of an Arch of 22 Degrees and ½ is equal to the exceſs whereby the Diagonal of a Quadrate exceeds the ſide of the ſame.

Let A B C D be a Quadrate, and in it a Quadrantal Arch Inſcribed B D, cutting the Diagonal A C in N. Then A N is equal to the ſide A B. I ſay N C is equal to the Tangent of an Arch of 22 Deg. ½.



Deſcribe the Quadrantal Arch A C, cutting the Diagonal D B in M: Alſo draw M N, it will be Parallel to the ſide B C, and M N being produced to both the ſides A B, D C, in L and O. Then D O will be equal to the Sine of an Arch of 45 Degrees, or to the

the half Diagonal DI. Let the Quadrate ABCD, be cut by the right Lines EF, GH, interſecting at I, four ways. Then is DO a mean Proportional between the whole ſide DC and his half DH. Wherefore as DC to DO, ſo is DO to DH; and likewiſe the difference CO to the difference OH.

Likewiſe CO and NO are equal, becauſe the Angles at C and N are half right Angles. Wherefore NC is in power double to CO; alſo CO is in power double to OH (being double in power, ſignifies that a Quadrate whoſe ſide is CO, contains as much again as a Quadrate whoſe ſide is OH); for when the ſide DC is in power double to DO, and DC, DO, OH continual Proportionals; CO is in power double to OH; therefore alſo HO, OC, NC are continual Proportionals, becauſe Nc is double in power to CO.

Draw the Chord DN; then the Angle ODN will be an Angle of 22 Deg. ½. Alſo the Chord DN cuts the right Line GH in R, and HR will be equal to HO.

Likewiſe becauſe RH, NO, NC are continual Proportionals in the Proportion of the ſide DC to DO, the Chord DN being produced to the ſide BC in P, will cut the part CP equal to the right Line NC, being equal to the exceſs of the Diagonal above the ſide. Wherefore a Tangent of an Arch of 22 Deg. ½, is equal to the exceſs of the Diagonal, *&c.* which was to be Demonſtrated.

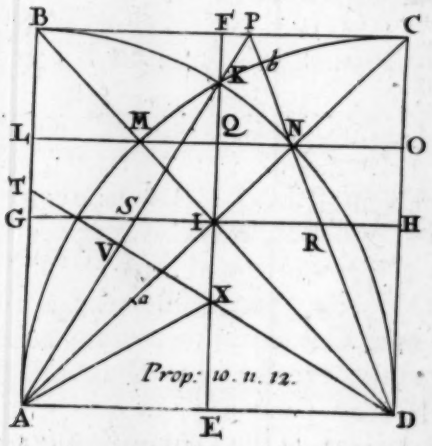
Cor. From hence it follows, that the right Line BP is equal to the double of CO. For if from the Centre C, with the Interval CD an Arch be drawn, cutting CA in *a*, *a*N will be double to IN, and A*a* equal to NC, therefore when as *a*C, and BC be equal, likewiſe *a*N, that is, the double of CO will be equal to BP.

PROP.

PROP. XI.

A Tangent of an Arch of 30 Degrees, together with a Tangent of an Arch of 22 Deg. ½, are equal to the side of a Quadrate BC.

Divide the Arch MC in the middle in *b*. Then either of the Arches C*b*, *b*M will be an Arch of 22 Deg. ½, and D*b* will pass by N. Also MK is a third part, that is, two sixths of the Arch CM. Then when as MK is a third part of MC, and M*b* an half, the Arch MK will be double to the Arch K*b*.



Then the Angle BDK is a sixth part of the Angle CDM, that is, a twelfth part of the right Angle.

Produce D*b* to the side BC in P.

Then because the Angle AKD is two thirds, that is eight twelfths of one right Angle, and the Angle KDP one twelfth; If AK be produced until it meet D*b* being produced, which will happen in P. For where-

whereſoever it cuts in D *b* being produced, it will make with it an Angle equal to ſeven twelfths of a right Angle, becauſe the Angle KAB is equal to eight twelfths, and the Angle KDP to one twelfth; for the remaining Angle KPD will be ſeven twelfths of one right Angle; for when the Angle CPD is nine, and the Angle which the Tangent of 30 Deg. makes with his Secant is eight, the remaining Angle will be a Complement to two right Angles, that is, to the three Angles BPA, APD, CPD. Therefore when as the Angle CPD is nine, and the Angle DAK eight twelfths, the other Angle APD will be ſeven, and all the three Angles together will be twenty four twelfths of one right Angle, that is equal to two right Angles, that is to the three Angles of the Triangle APD. Wherefore the Tangent of 30 Deg. *&c.* which was to be Demonſtrated.

ANIMADVERS.

This Prop. hath long ſince been confuted by Tables of Sines, Secants and Tangents, calculated from ſeveral Geometricians by Dr. Wallis.

Whether Mr Hobbs *or he is to be credited, is left to the Readers conſideration.*

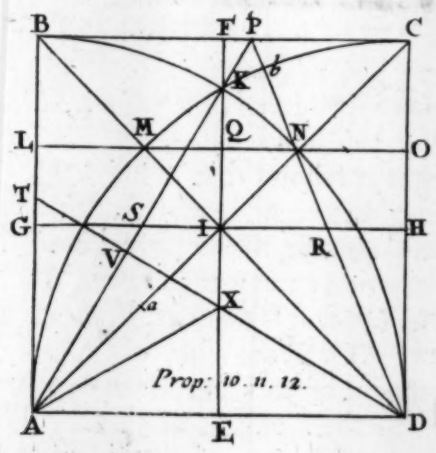
From hence it follows that BM, MN, NC, CP, KS, are equal between themſelves. For whereas every one of them is double to the right Line HO or IQ, they will be equal among themſelves. Beſides BP is double to GS, for it is manifeſt from this that AB is the double of AG. From whence it appears again that BP is a Tangent of 30 Deg. for AS, which is manifeſtly equal to the Tangent of 30 Deg. is the double of GS.

PROP.

PROP. XII.

A right Line which cuts the Base AK *of an Equilateral Triangle from any Vertical Point in the middle, is Sesquialter of the Tangent of an Arch of* 30 *Degrees.*

LEt there be taken in AB, a Tangent of an Arch of 30 Deg. AT. Joyn DT cutting AK in V, and the right Line EK in X, and draw AX. Then the Triangle ATX will be Equilateral, and both the Angles AVD, and KVD are right Angles; likewise the sides AX, XD will be equal. And TX is the double of VX. Wherefore DV is the Triple of VX, that is, Sesquialter (*viz.* once and an half) of DX, that is, a Tangent of 30 Deg. which was to be Demonstrated. The same Proposition is Demonstrated



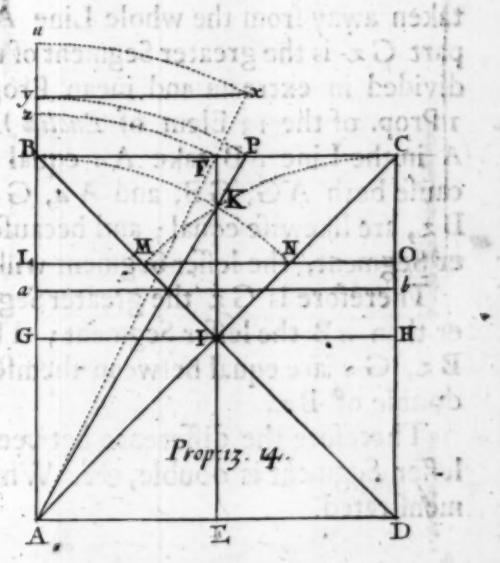
by *Euclid* in his 18 Prop. of the 14 Elem. but becauſe it is too long to rehearſe in this place, let it be read qy him that doubts of the truth of it.

Corol. Therefore the right Line DV or EK, is Triple the difference between the ſide DC and the half Diagonal DI or DO; for CO is manifeſted to be equal to half the Tangent AT.

PROP. XIII.

The difference between the greater and leſſer Segment of the right Line divided in extream and mean Proportion, is double the difference between the ſame right Line, and a right Line whoſe power is to it, as 5 to 4.

LEt the given right Line be AB, to which let there be put at right Angles BF, half of the given Line AB, then draw AF, the power whereof, is to the power of the given Line, as 5 to 4, and to the power of BF, as 5 to 1.



Prop. 13. 4.

Note, That the word *power* will be often

uſed, therefore by the *power of a Line*, you are to underſtand a Quadrate or Square made from the Line mentioned, each ſide whereof is equal to the mentioned Line, as if it had been ſaid, a Quadrate or Square drawn from AF (that is, a Quadrate, each of its ſides being equal to AF) is to the Quadrate drawn from the given Line AB, as 5 to 4, and to the Quadrate drawn from BF, as 5 to 1.

With the Interval AF, deſcribe an Arch of a Circle, cutting AB being produced in z. Then the Square from the right Line Az, is to the Square made from the given Line AB, as 5 to 4, and to the Square from BF, as 5 to 1.

I ſay the difference between the greater and the leſſer Segment of the given Line AB, divided in extream and mean Proportion, is the double of Bz.

Divide AB in the middle in G; then AG being taken away from the whole Line Az, the remaining part Gz is the greater Segment of the given Line AB divided in extream and mean Proportion, (by the 1 Prop. of the 13 Elem. of *Euclid*). From the Point A in the Line AB take A*a* equal to Gz; then becauſe both AG, GB, and A*a*, Gz are equal, G*a*, Bz, are likewiſe equal; and becauſe A*a* is the greater Segment, the leſſer Segment will be *a*B.

Therefore is Gz the greater Segment, being greater than *a*B the leſſer Segment; The two right Lines Bz, G*a* are equal between themſelves, that is, the double of Bz.

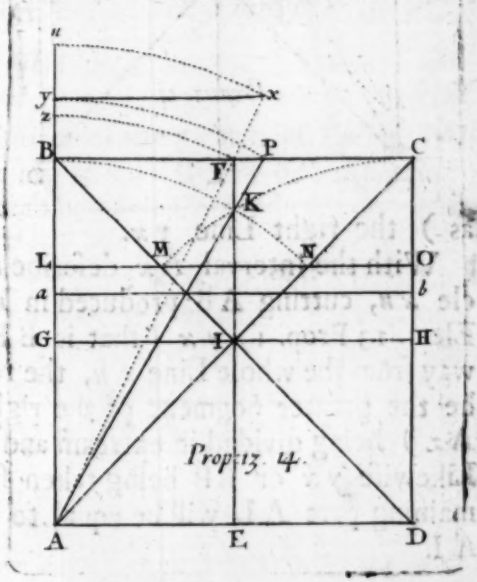
Therefore the difference between the greater and leſſer Segment is double, *&c.* Which was to be Demonſtrated.

PROP.

PROP. XIV.

If the Secant of an Arch of 30 Degrees be cut in extream and mean Proportion, the greater Segment will be equal to the Semidiagonal of a Quadrate made from the Semidiameter.

DEscribe a Quadrate from AB (to wit) ABCD, and divide it into 4 parts by the right Lines EF, GH, also divide it into 4 parts by the Diagonal Lines AC, BD, all concurring in the Centre of the Quadrate at I; let there be described two Quadrantal Arches AC, BD, cutting the Diagonals in M and N, and the right Line EF in K, draw AK and produce it to the side BC in P. Then AP is the Secant of the Arch BK, which is an Arch of 30 Degrees; likewise BP is a Tangent of 30 Deg. and the half of the Secant AP.

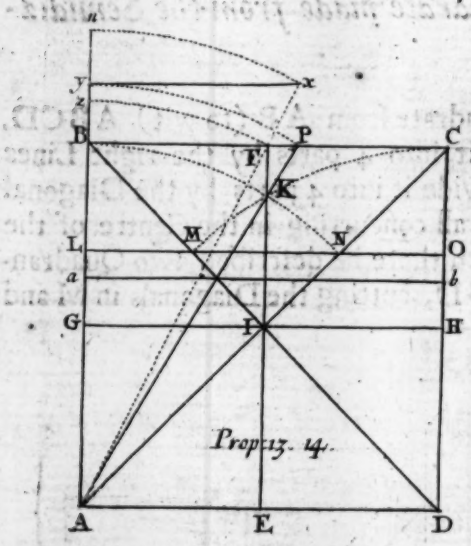


Prop. 13. 4.

By the Points M and N, draw the right Line

LO equal and parallel to the side BC Wherefore AL, or DO is equal to the Semidiagonal AI.

I say AL is the greater Segment of the Secant AP divided in extream and mean Proportion.



Prop. 13. 14.

With the Interval AP describe an Arch of a Circle P*y*, cutting AB produced in *y*.

From the Point *y* draw *yx* parallel to the side BC, & equal to half the Secant AP, that is, equal to the Tangent BP. Then A*x* being drawn, is in power Quintuple to (or five times as much as) the right Line *yx*.

With the Interval A*x* describe an Arch of a Circle *xu*, cutting AB produced in *u*. Wherefore (by Elem. 13 Prop. 1) *yx* (that is BP) being taken away from the whole Line A*u*, the remaining part will be the greater Segment of the right Line AP (or A*z*) being divided in extream and mean Proportion. Likewise *yx* or BP being taken from A*u*, the remaining part AL will be equal to the Semidiagonal AI.

For

For it is manifeſt by Prop. 10, that the right Line B P is double to the right Line C O or B L. Therefore *v* L is equal to *y x* or B P; and the remaining part A L the greater Segment of the Secant A P, or the right Line A *y* divided in extream and mean Proportion. Which was to be Demonſtrated.

Corol. From hence it follows, that the half Diagonal A I is the greater Segment of half the Secant A P, that is, of a Tangent of 30 Deg. that is, of a ſide of a Cube inſcribed in a Circle, whoſe Diameter is A B.

Prop. XV.

A Digreſſion concerning the Diſcord between the Computation of Lines, of Superficies, and of Numbers in the Demonſtrations of Geometricians.

By the 5 Definition of the 5 Elem. of *Euclid*, Magnitudes are ſaid to have Reaſon or Proportion to one another, which being multiplyed may exceed one another.

From which Definition it is manifeſt, that Lines, Superficies, and Solids can have no Proportion among themſelves. For being multiplyed together in themſelves, they can never mutually exceed.

If notwithſtanding for a Line, there be uſed a ſmall right Angle, there may truly ſometimes a Proportion be found between a Superficies and that right Angle, though it be very ſmall; to wit, when two Quadrates are to each other as a Quadrate Number to a Qua-

drate Number. And one is the measure of the other, because they may be compared.

But the Quadrates which are between themselves as Quadrate Numbers, are much fewer than those which are not as Quadrate Numbers, although both are innumerable.

Therefore the Doctrine concerning the Quantity of Lines, is a Science subsistent by it self, and distinct from the Science of Superficies, and this distinct from the Science of Solids.

Moreover because all mensuration begins from a Point, and a Point cannot be considered as Figurative, how can a Point in a Quadrate Angle any otherwise be considered, than as a *quantum* common to the Quadrate and side of it? For to consider it as Figurative and not Figurative, is absurd.

Besides how can a Point, which is in the Centre of a Circle, be accompted for nothing, when as it is divisible? For in how many parts soever any Sector is divided, in so many parts likewise is the Centre divided.

Perhaps some may say, without the known quantity of Figures, very few of Theorems hereafter concerning the Proportions of Lines are Demonstrable, besides Mechanick mensuration. But they err, First, because the Proportions of Lines between themselves, and the Construction of Figures, and all their affections of motion are delivered by *Euclid*, without the known quantity of a Quadrate, or of any other Figure, or the Proportion of Figure to Figure. Neither hath any one attempted to Demonstrate the length of a Line, by the magnitude of a Quadrate before *Archimedes*; neither after him (that I know) besides *Eutocius*, before *Copernicus*, truly neither hath he Demonstrated accurately.

Neither

Neither do I ſay theſe things, becauſe I would admit Mechanick Operations for lawful Demonſtrations. But in every Geometrical Queſtion, I think it much more prudence before Mechanically meaſuring the magnitude ſought, as much as may be done with truth to attempt the neareſt way, and afterwards enquire into the cauſe of that nearneſs, which being found out will detect the truth or falſhood; than raſhly believing of incertain reaſoning, or the reckoning of thoſe, or the authority of others, who pronounce unknown things, eſpecially not only where the certainty of one, but of many Theorems are deſtroyed. A diligent Meaſurer declares more credible things by meaſuring, than he that reaſons from falſe Principles, and will deſervingly ſcorn the Algebraiſts, that is, the Arithmeticians diſputing againſt meaſure, and ſaying, that the ſide of a Quadrate, and the root of a Number are the ſame.

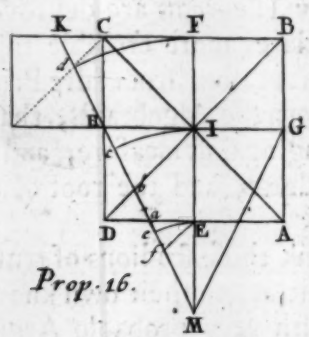
Alſo no man will think thoſe ſtudious of truth, who ſeeing a Concluſion contrary to their own knowledge, although ſupported with very probable Arguments, are content to reſiſt the ſole Demonſtration, and neglect the verity of the thing (which ſometimes proceeds from the debility of their own wit, or from the omiſſion of ſome Propoſition, which the Demonſtrator ſuppoſed to be well known to Geometricians, eſpecially the chiefeſt of them) for theſe kind of men are not ſeeking the truth but victory.

What I have ſaid concerning the incongruity of Lines and Superficies, will appear clearly in the following Problem, being propoſed to Algebraiſts.

PROP. XVI.

DEscribe the Quadrate ABCD, and divide it (both) with the right Lines EF, GH, and with the Diagonals AC, BD, interſecting in I four ways.

In FC being produced, take FK equal to the half of MF, and joyn MK, then with the Radius MF, deſcribe an Arch F*d*, it will cut in *d*, ſo that MF, M*d* are equal. And MK cuts the right Line E*d* in *a*, and the Diagonal ID in *b*. Alſo MK will paſs by H.



Prop. 16.

Draw E*f* Parallel to ID cutting MK in *f*. Moreover with the Radii ME, MI, deſcribe the the Arches E*e*, I*c* cutting MK in *e* and *c*.

It is manifeſt from this Conſtruction; Firſt, that M*a*, *a*H, HK, are equals between themſelves.

Secondly (becauſe MI is double to ME) M*b* is double to M*f*.

Thirdly (becauſe ME is the double of E*a*) M*f* is the double of *fa*; and (becauſe MI is double to IH) H*b* is double to *ba*; and then it will be as H*a* to H*b*, ſo M*a*, to M*f*, as 3 to 2, or 9 to 6.

Fourthly, It is manifeſt, that as well K*d* as *ba* is to *ae*, as 3 to 1, and to H*c* as 3 to 2, and ſo likewiſe *fe* will be to *ae* (to wit) as 3 to 2, or 9 to 6, and

and therefore C*d* being joyned is parallel to ID; therefore *d*H, H*b* are equal.

Fifthly, It is manifeſt, that M*f*, *fb* are equal, and *b*K, *da*, are equal

Then M*d* is four times two, whereof MK is thrice three.

Wherefore MK is to M*d* or MF, as 9 to 8.

Then becauſe MK is Quintuple in power to FK or M*b*, if FK be taken away from the right Line MK; the remaining part *b*K (by Elem. 13 Prop. 1) will be the greater Segment of MK, or *bd* of M*d* being divided in extream and mean Proportion.

But *b*K is equal to *da*. Wherefore *da* is the greater Segment of M*d* divided in extream and mean Proportion.

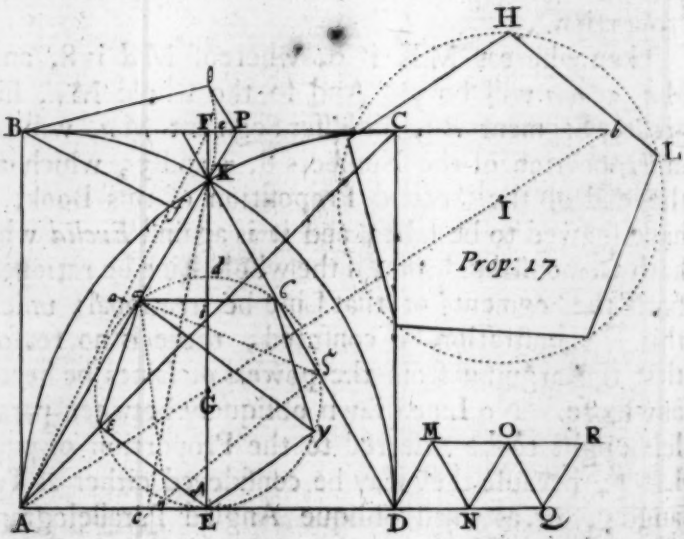
Then whereas MK is 9, whereof M*d* is 8, and M*a* 3, *da* will be 5. And ſo the whole M*d*, his greater Segment *da*, his leſſer Segment M*a*, will be in Proportion of the Numbers 8, 5 and 3; which at the end of the Second Propoſition of this Book, I have ſhewed to be falſe; and it is againſt *Euclid* who hath Demonſtrated, that if the whole Line be rational, both the Segments of that Line be irrational; unleſs this Demonſtration be confuted, there is no reaſon that the arguing from the powers of Lines be heard any more. No Line drawn obliquely between parallels ought to be referred to the Proportion of pure Lines; becauſe they may be conſidered either as Triangles, or as ſmall oblique Angled Parallelograms whoſe Longitude is not determined. For the Longitude of a Figure is nothing ſurely beſides that which is called Altitude.

PROP.

PROP. XVII.

The side of an Icosahedron is equal to the third part of the greatest Semicircle in its own Sphear.

DEscribe the Quadrate ABCD and divide it in the middle by the right Line EF parallel to the sides AB, DC, also describe a Quadrant ABD whose Arch will cut the right Line EF in K. Joyn AK



being produced to BC in P. Then is BP a Tangent of 30 Degrees, to which add P*b* equal to the side BC, the whole Line B*b* will be compounded of the side, and a Tangent of 30 Deg. Likewise draw the Diagonal

Diagonal AC, which the right Line EF will cut in the middle in *i*.

Joyn DK, and AKD will be an Equilateral Triangle.

In the ſide AD take A*c* a third part of it, and joyn *c*K, and produce it to the ſide B*c* in *e*; and F*e* will be a third part of FP, and therefore B*e* will be a third part of the whole Line B*b* (by Conſect. 3 of the 7 of this.)

In the right Lines KA, KD, let there be taken K*α*, K*ζ* either of them equal to a fourth part of the Diagonal AC, or the half of A*i*, and joyn *α ζ*. Then the Triangle K*α ζ* will be Equilateral; and its Baſe *α ζ* will be cut by the right Line KE in the middle, and at right Angles.

Of the right Line BP, which is a Tangent of 30 Degrees, and of two right Lines, whereof either is equal to *α ζ*, make the Triangle *α ζ γ*; and *ζ γ*, *α ζ*, will be equal.

By the three Points *α*, *ζ*, *γ*, deſcribe a Circle, whoſe Centre is G, and Semidiameter G *ζ* or G*α*.

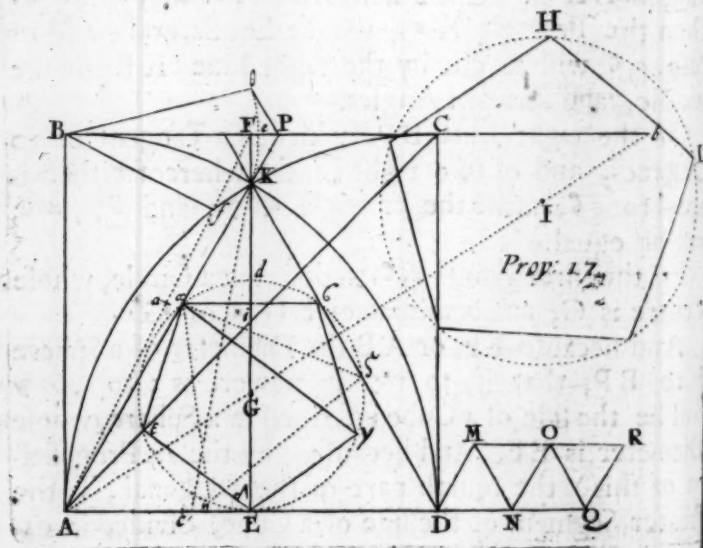
And becauſe EF or AB the Diameter of a Sphere is to BP, that is, to *α γ* in power, as 3 to 1, *α γ* will be the ſide of a Cube inſcribed in a Sphere, whoſe Diameter is EF. And becauſe (by the 14 Propoſition of this) the fourth part of the Diagonal, is the greater Segment of the ſide of a Cube, divided in extream and mean Proportion; it will be (by the 8 Prop. of the 13. Elem.) that the right Line *α ζ* is the ſide of a Pentagon in the Circle *α ζ γ*; and that ſide of a Pentagon one of the twelve Seats or Points of a Dodecahedron inſcribed in the ſame Sphere with the Icoſahedron.

Compleat the Pentagon *α ζ γ δ ε*.

Draw

Draw the right Line AF cutting the Arch AKC in f; and the power of Bf will be (as is well known to Geometricians) a fifth part of the power of the ſide AB, or Quintuple in power to the Diameter of the Sphere.

Deſcribe a looſ off with the Radius IH, being equal to Bf, the Circle HL, in which the ſide of an Equilateral Pentagon is HL. Then HL will be the ſide of an Icoſahedron in the ſame Sphere, by the 16 Prop. of the 13 Elem.



Then (by the 5 Prop. of the 14 Elem. of *Clavius's* Edition) the right Line HL is the ſide of an Equilateral Triangle inſcribed in the ſame Circle. Inſcribe in the Circle $\alpha\gamma\delta\varepsilon$, the Equilateral Triangle $\alpha\zeta$. Then $\alpha\zeta$ will be the ſide of an Icoſahedron in the Sphere whoſe Diameter is EF.

It is to be ſhewn that the right Line $\alpha\zeta$ is equal to a third part of the Semicircle whoſe Semidiameter is BF, that is to the Arch BK, which is a third part of a Quadrant of a Circle deſcribed from AB as a Semidiameter.

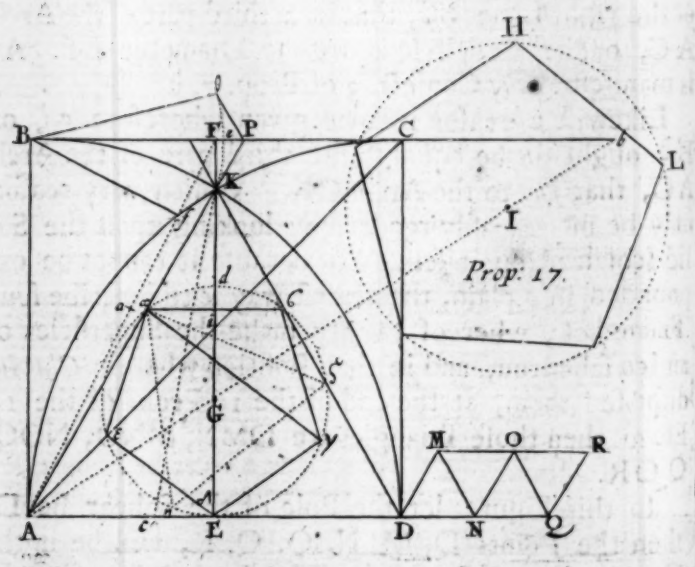
With the Semidiameter Be, deſcribe an Arch $e\theta$, in which take $e\theta$ equal to the Arch $\gamma\zeta$. Then draw Bθ it will be equal to $\alpha\zeta$, that is, to the ſide of an Icoſahedron in a Sphere, whoſe Diameter is EF. Then with the Radius Bθ, deſcribe an Arch of a Circle, cutting BP, it will give a ſide of an Icoſahedron, which ſide if it be the ſame Be, will be a third part of the right Line Bb, that is, a third part of the Arch AC, or the Semicircle above the Diameter EF. As is manifeſted by Conſect. 3 of Prop. 7.

Likewiſe a reaſon is to be given wherefore $\alpha\zeta$ or Bθ ought to be equal to the third part of the Arch AC, that is, to the Arch CK. Which very reaſon may be more eaſily rendred by looking upon the Solid Icoſahedron it ſelf. But becauſe it cannot be expounded in a plain, the neareſt way is to deſcribe four Triangles, whereof twenty make the Superficies of an Icoſahedron, and in that Poſition wherein *Clavius* diſpoſed them, at the end of the 16 Prop. of the 13 Elem. then thoſe Triangles are DMN, MNO, NOQ, QOR.

In this Figure let the Pole of the Sphear be D, then the Points D, M, N, O, Q, R, will be in the Concave Superficies of the Sphear; and therefore the right Lines MN, NO, OQ, will not be in the ſame Plain with the Points D and R, which are in the Plain ſtreight by the Diameter of the Sphear.

Wherefore the ſide of an Icoſahedron proceeds from the Pole D to the Pole R, by five equal right Lines,

Lines, to wit, from D to M, from M to N, from N to O, from O to Q, from Q to R. And first in the motion from D to M, it is moved forward towards R. Again from M to N, it is not moved forward towards R. Thirdly from N to O, it is moved forward, as much as from D to M. Fourthly from O to Q it is not moved forward towards R. Fifthly from Q it is moved forward to the very Point R. Therefore by five equal right Lines, which are the sides of an Icosahedron, is the motion made from Pole



to Pole, but by reason of the digression to the nearest Circles which divide the Superficies of the Sphear in five parts, the motion is made by the five sides of an Icosahedron.

But in the circumference of a Semicircle, the motion is made from Pole to Pole by three Arches, every one

one of them being equal to the Arch CK or BK. The motion therefore being taken away, which is made by thoſe two ſides of an Icoſahedron which move not forward, the ways by the three ſides of an Icoſahedron and by three Arches of a Semicircle, every one of which being equal to the Arch CK, or at leaſt by the Chords of thoſe Arches will be equal. But B0 will be found greater than the Chord BK. Wherefore the Arches every one being equal to BK, are equal to three ſides of an Icoſahedron, and one to one. Which was to be Demonſtrated.

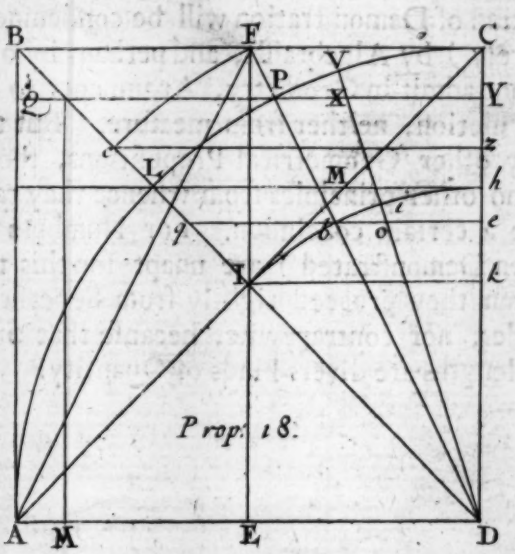
This kind of Demonſtration will be condemned (I know ſurely) by Algebraiſts, and perhaps by others, who do not admit in Geometry, Arguments to be taken from motion, neither from meaſure. But to this and many other Geometrical Propoſitions, they can find out no other principles from whence they can truly derive a certain concluſion. For Numbers (as I have often Demonſtrated) are unapt for this thing, neither can they proceed rightly from Superficies to Longitudes, nor contrarywiſe, becauſe that Superficies and lengths are divers kinds of Quantity.

PROP.

PROP. XVIII.

A Circle being given to find a Quadrat equal to it.

LEt the Circle given, whose Quadrant is DAC, be inscribed within a Quadrat ABCD; and DLC the eighth part of the Circle. Cut the Quadrat ABCD with the Diagonals AC, BD; also with the right Lines EF, Ik, all Intersecting in I,



and dividing the Quadrat into four parts both ways; and draw DF cutting the Arch CL in P, and by the Point P draw YQ, cutting the Diagonal BD in Q, then DY, YQ will be equal.

With the Radius DF, describe an Arch F *c* cutting the Diagonal BD in *c*, so that DF, D*c* may be equals.

I ſay the Quadrate from YQ or DY is equal to an eighth part of the Superficies of the Circle DCL.

From the Point *c* draw *cz* parallel to YQ cutting DC in *z*. Then is *zc* (by the 6 Prop. of this) half the Arch AC, and therefore equal to the Arch CL.

In the Arch CL, take the Arch LV equal to CP, and joyn DV, cutting YP in X; and the three Lines CYP, will be wholly within the Sector DCL; but the three Lines PQL are all without the ſame Sector.

Likewiſe both the three Lines together are (as before I have ſhewn, and now will ſhew) equal to the Sector APV. For becauſe the right Line BC is cut in the middle at F, and the Baſes of the Triangles DCB, DYQ are parallel; alſo the Baſis YQ is cut in the middle in P, and the Triangles DYP DPQ are equal.

Now DPL more PQL more CPY are equal to DVL, or DCP (becauſe DPL more PQL is equal to DYP.) For DCV more DVP is equal to DCP, or DVL.

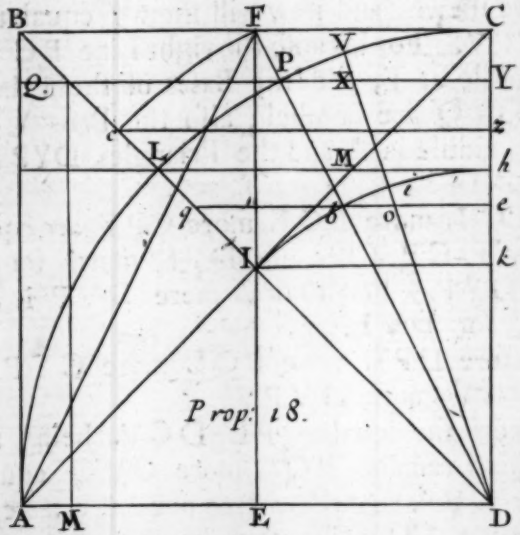
Wherefore DPL more PQL more CYP is equal to DCV more DVP.

Then both the equals DPL, DCV, being taken away, there remains PQL more CYQ equal to the Sector DVP. And hitherto our Adverſaries conſent or agree. This alſo they muſt condeſcend to, (for it is manifeſt) that if CYP, PQL, are equal between themſelves, the Triangle DYQ, and the Sector DCL are likewiſe equal.

And I had thought it Demonſtrated before from this, becauſe the Triangle whoſe Vertex is D, and Baſis parallel to the ſide BC, no Triangle can be made equal to the Sector DCL, whoſe Baſe paſſeth not

by P; but the force of the Demonſtration they have not clearly underſtood. Go too then, we will produce if it be poſſible, a more clear Demonſtration.

With the Radius D I, which is a mean Proportional between the ſide D C, and the half ſide D *k*, deſcribe an Arch of a Circle cutting D F in *b*, and D V in *i*, and D C in *h*; and by the Point *b* draw *e q*, cutting D C in *e*, and the Diagonal B D in *q*, and D V in *o*. Then the Sector D C P will be double



Prop. 18.

of the Sector D *h b*, as alſo of the four Lines C P *b h*; likewiſe the Triangle D X Y double of the Triangle D *o e*, it is likewiſe double of the four Lines Y X *o e*. Wherefore the remaining Sector D V P is double to the three Lines C Y P, and likewiſe double to the four Lines V P *b i*. And D *b i* double to the three Lines *h b e*.

Then

Then when as the Sector DVP is double to the three Lines CYP, and the ſame equal to two three Lines, CYP, PQL; then CYP, PQL will be equal between themſelves.

Which was to be Demonſtrated.

Therefore a Quadrate is found (to wit a Quadrat from YQ) equal to the eighth part of a Circle, to wit, to the Sector DCL; and ſo is effected the Squaring of a Circle, nor now the firſt time, but many years ago ſufficiently Demonſtrated by divers methods.

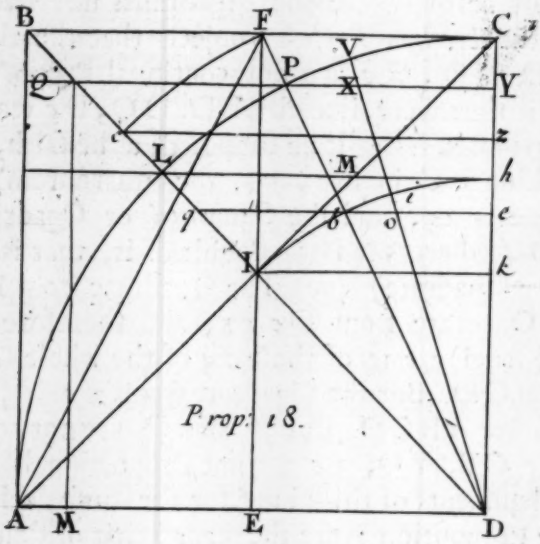
From this Demonſtration likewiſe I have deduced the duplicating of a Cube, ſhewing that the four right Lines CB, *zc*, *eq*, and *Ik* are continual Proportionals. But neither have the Algebraiſts underſtood this; for the *Savilian Profeſſor* objects that the right Lines CB, YQ, *zc* are not continual Propottionals. For if they were, likewiſe BD, DQ, D*e* would be Proportionals. Let it be ſuppoſed, as he ſaith, divide the Line BC in five parts, the Square from the whole Line is 25, and the Square (or Quadrat) from BD (whereas it is the double of it, that is, of the former Quadrate) 50. The Quadrate from DQ 40, the Quadrate from D*c* 32, and therefore the Quadrate from *zc* 16 of thoſe 25 of the whole Quadrate ABCD. But the Quadrate from *zc* is $\frac{16}{25}$ of the Quadrate ABCD. But $\frac{16}{25}$ and $\frac{10}{16}$ are not equal. Therefore CB, YQ, *zc* are not Proportionals.

But Arguments of this kind (for the cauſe declared at the 15 Propoſition) are the meer Spirit of deluſion in Algebraiſts, in applying thoſe Numbers to Quantities, which have not the Proportion of Number to Number.

If BC be divided in five parts, the Quadrate from DF will be $6\frac{1}{4}$, to wit, the fourth part of 25. Where-

fore D*c* will be 31 $\frac{1}{4}$, and the Quadrate from BD will be eight times as much as the Quadrate from BF, or 50.

But the mean Proportional between 8 and 5, will be the ſide (I call it the ſide not the root) of the number 40. Then not only 50, 40, 32, but alſo 50, 40, 31 $\frac{1}{4}$ will be continual Proportionals. Therefore what the cauſe of this diſcord ſhould be (ſince it is plainly Demonſtrated, that both *z c* is equal to half the Arch AC, and that the Quadrate from YQ is equal to the Sector CL) unleſs that Lines drawn (that is diviſible according to breadth) cannot be compared



with pure Lines, that is, without breadth. But the right Lines CB, *z c*, *e q*, *k* I are continual Proportionals, as I ſhall make appear moſt clearly in the following Demonſtration.

CON-

CONSECT. I.

The Arch CL is leſs than four fifths of the Radius DC; for all the verſed Sines which are poſſible to be drawn in the Quadrant DAC, being taken together are equal to the Area or content of the Quadrant, and by conſequence to the Quadrat DYQM, or to four fifths of the Quadrate ABCD. And for becauſe thoſe verſed Sines are all bounded within the Arch, the two ſides BC, CD of the Quadrat ABCD ought to be to the halfs of the Arch AC, in Proportion double of CD to DY; but it is not ſo. For although you ſhould divide Lines or other continued Quantity infinitely, notwithſtanding you will never attain to nothing; becauſe continual quantity is always diviſible into diviſibles. Then thoſe Sines, whereof the whole number of them fill the Area of a Quadrant will have every one their breadth, and will always be finite in number. Alſo thoſe verſed Sines are parallels between themſelves, whereof the greateſt DC is the Radius of a Circle whoſe bound (or end) at C is a ſmall Arch. Alſo DC is the ſide of a Quadrat ABCD, and therefore (becauſe it hath breadth) it will be a right Angle, and therefore more than the Radius DC. And part of it will be without the Circle. The Proportion of the reſt of the verſed Sines is the ſame; and for becauſe according to breadth, they may be divided as often as it is poſſible to divide, they will never proceed to an indiviſible.

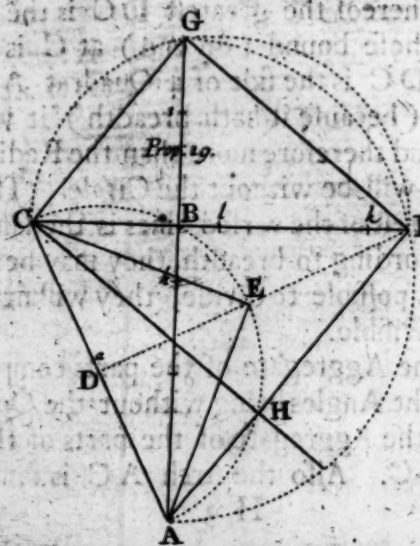
Therefore the Aggregate of the parts compaſſed about of the right Angles being without the Quadrant, is more than the Aggregate of the parts of the Quadrant it ſelf AC. Alſo the Arch AC is ſome ſmall

matter less than four fifths of the Diameter, and the Arch CL, or the right Line z c less than four fifths of the side DC.

PROP. XIX.

Between a right Line given, and the half of it; To find two mean Proportionals.

LEt the given Line AB and the half of it BC be disposed at right Angles; joyn AC, and let AC be cut in the middle in D; from the Centre D with the Radius DA, describe a Semicircle, and divide it in the middle in E. Draw AE, CE, and in CB being produced, put BF equal to CE. In AB being produced, take two fifths of the right Line AB, which let be Bi, and between AB and Bi



find

find a mean Proportional B*k*: Equal to it in AB produced, put BG (which as above is Demonstrated, is equal to two fifths of the Quadrantal Arch described from AB) and draw FG, GC. Moreover divide CF in the middle in *l*, from the Centre *l* with the Radius *l*F, describe a Semicircle by F and G, which will pass by C; which I shew thus.

The right Line AC, whose Quadrate is Quintuple of the Quadrate BC, is a mean Proportional betwixt the Quadrantal Arch described from AB and its half, as is shewn at Prop. 6. Therefore CB is equal to half the Quadrantal Arch described from AB (for because the Angle by E is a right Angle, CE is a mean between both the right and Circular Lines CE, EA and either of them. Wherefore BF is equal to half the Quadrantal Arch described from AB. And BG is a third continual Proportional to BF and the whole Line AB by Prop. 7. Then because the Quadrantal Arch from AB, and the right Line AB, and two fifth parts of the Quadrantal Arch from AB, are continual Proportionals; it will be, as two fifths of the Quadrantal Arch from AB (or the right Line B*k*) to the half AB so AB to the half of the Quandrantal Arch from AB. Therefore it is as AB to BF, so BG to BC.

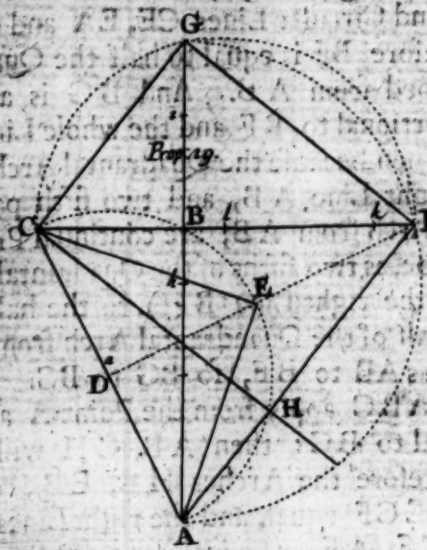
In the Arch ABC apply from the Point A a right Line AH equal to BC; then AB, CH will be equal, and therefore the Arches HE, EB, or the Angles HCE, ECF equal, and the right Lines HF, BF, likewise CF, AF are equal, and DE produced, divides the Angle AFC in the middle.

Also the Triangles ABF, GBC are alike, wherefore the Angles BGC, BAF are equal, also the Angles BCG, BFA equal. And (for because the Angle CHA is a right Angle, and CH equal to AB) the Tri-

angles ABF, FBG are alike and equal. Therefore both BF, GC, also CH (or AB) and FG are equal between themselves.

And AG being divided in the middle in *h*, from the Centre *h*, with the Radius *h*A a Semicircle being described it will pass by F.

Then is FGCH right Angled, and the Triangles ABF, FBG alike. Therefore it is as AB to FB, so FB to BG, and so BG to BC. Therefore BF, BG are found to be two mean Proportionals between the given Line AB, and the half of it BC. Which was to be done.



Cor.

It is manifest from the precedings, that the greater of the two means is an eighth part, and the lesser mean two fifth parts of the Quadrantal Arch described from

from A B. Alſo A B (or C H) is equal to F G; and B G is equal to that part of the right Line C H, which is cut off by A B being computed from the Point C, and the Conſtruction of the Problem Demonſtrated to the Eyes of all Men.

PROP. XX.

Of the Centre of Gravity of a Quadrant of a Circle.

THe Centre of Gravity (that is, the Centre of weightineſs or heavineſs) of a Quadrant of a Circle is in a right Line from its Centre, dividing the Arch in the middle, and diſtant from the Centre of the Circle, ſo much as is the mean Proportional between the Semidiameter, and two fifth parts of it.

Deſcribe the Quadrate A B C D, and in it a Quadrant A D C, and draw the Diagonals A C, B D, whereof B D will cut the Arch A C in the middle in L.

Between the Semidiameter D C, and two fifth parts of it, find a mean Proportional D R, and with the Radius D R, deſcribe an Arch of a Quadrant R S, cutting the Diagonal B D in z. I ſay z is the Centre of Gravity of the Quadrant D A C.

Cut the Quadrate A B C D into four parts by the right Lines E F, G H, cutting themſelves mutually and at right Angles in I. Joyn D F cutting the Arch A C in P; and by P draw Y Q parallel to B C, cutting the Diagonal B D in Q, and E F in V; alſo compleat the Quadrate D Y Q M, whoſe ſide Q M cuts the Arch A C in N.

Then as the Quadrate from D F, to the Quadrate from

from DP or DC, so is the Quadrate from DC to the Quadrate from DY. And the Quadrate ABCD is 5 whereof the Quadrate DYQM is 4.

Joyn YM, and it divides DQ in the middle; also draw the right Lines Pk, NO, that Parallel to the side AB, this parallel to the side BC, they will cut each other mutually, and at right Angles in the middle of the right Line DQ.

Therefore because the Quadrate ABCD is to the Quadrate DYQM as 5 to 4, the Quadrate of DQ will be 8, whereof the Quadrate from YQ is 4, and the Quadrate from DC 5, and the Quadrate from the half DQ 2.

Then the Quadrate from the half of DQ, is two fifths of the Quadrate ABCD; therefore the half of DQ is a mean Proportional between the Semidiameter DC, and two fifths of it, and therefore equal to Dz. Likewise z is both the Centre of magnitude, and also the Centre of Gravity of the Quadrate DYQM; and the Point I the Centre of magnitude and of Gravity of the Quadrate ABCD.

Also it is shewn at the 18 Prop. of this, that the Quadrate DYQM, and the Quadrant DAC between themselves are equal. And that the three Lines CYP, PQL, AMN, NQL between themselves are equal. Therefore if from the Quadrate DYQM be taken two three Lines equal to PQL, NQL, and to the same Quadrate be added two three Lines CYP, AMN at equal distances from the Diameters of even height, the three Lines CYP, AMN, will be equiponderant (or weigh alike) but the distances YP, MN, PQ, QN are equal and alike distant from the Diameters of equal height NO, Pk; likewise the Points Y, Q, M, D are equally distant from the Centre

tre of Gravity of the whole, z; therefore z is the Centre of Gravity of the Quadrant DAC. Which was to be Demonstrated.

CONSECT.

The Centre of Gravity of a Semicircle, is a mean Proportional between two fifths and one fifth of the Arch AC. For it is shewn (Prop. 5) that the right Line which is a mean between the Semidiameter and two fifths of it, is equal to two fifths of the Arch AC. Therefore if zO be produced to p, so that zO, Op be equal, then draw Dp, p will be the Centre of Gravity of a Quadrant equal to DAC. And because the Point O divides zp in the middle, the Point O will be the Centre of Gravity of the double of the Quadrant DAC, that is of a Semicircle described with the Radius DC, also Dp produced, cuts the Arch Cl equal to the Arch LC. Then when as the Quadrant DAC resteth in z, and the double of the Sector Cl resteth in p, the whole Semicircle will rest in O. But zO is a mean Proportional between Dz, and its half, that is between two fifths, and one fifth of the Arch AC, because the Triangle DOz is right Angled and Equicrural; (*viz.* it hath two sides equal.)

PROP.

PROP. XXI.

The Centre of Gravity of the Circular Line and ſtreight Line ALCA *is in the Diagonal* BD, *diſtant from the Point* B, *ſo much as is the length of a Tangent of* 30 *Degrees.*

FInd a Tangent of 30 Deg. BX, equal to which, from the Point B in the Diagonal BD take BT. I ſay that the Point T is the Centre of Gravity of the two Lines ALCA.

For becauſe the Quadrat from AB, is to the Quadrat from BX, or BT, as 3 to 1, and the Quadrat from the Semidiagonal BI is half of the Quadrat from AB, the Quadrat from BI will be to the Quadrat from BT, as $\frac{1}{2}$ to 1, that is, as 3 to 2.

And the three Lines ABCLA, to the two Lines ALCA, as 2 to 3. Which I thus ſhew.

The Gnomon YBM, is a fifth part of the Quadrat DYQM, that is, a fifth part of the Quadrant DAC. Wherefore alſo the three Lines ABCLA is a fifth part, or two tenth parts of the Quadrat ABCD. And the Triangle ABC is the half, or five tenths of the Quadrat ABCD. But the Quadrant DAC is four fifths, or eight tenths of the Quadrat ABCD. Therefore the two Lines ALCA is three tenths of the Quadrat ABCD.

Then is the Proportion of the three Lines to the two Lines, the ſame which 2 is to 3, that is, reciprocal of the Proportion, as well of the magnitudes ALCA, ABCLA, as alſo of the Quadrates BI, BT.

Joyn

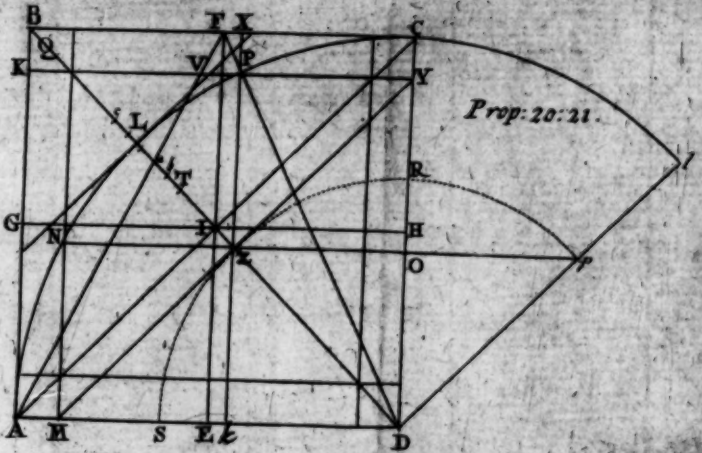
Joyn AF cutting the Diagonal BD in *a*. Then because AB is double to BF, and the Angle ABF divided in the middle by the right Line B*a*; A*a* will be the double of *a*F. Therefore the Point *a* is the Centre of Gravity of the whole Triangle ABC.

Cut *a*T in the middle in *b*, and take *a c* the Triple of T*b*, hnd let *a* be the Centre of the Ballance. It will be therefore as the three Lines ABCLA, to the two Lines ALCA, that is, as 2 to 3, so reciprocally *c a*, to *a*T, to wit, as 3 to 2.

Therefore is T the Centre of Gravity of the two Lines ALCA; and the Point *c*, the Centre of Gravity of the three Lines ABCLA. Which was to be Demonstrated.

The End of the Second Book.

SOME



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SOME

PRINCIPLES AND PROBLEMS IN GEOMETRY

Thought formerly Desperate;

NOW

Briefly Explained and Demonstrated.

The Third BOOK.

Written in Latin

By *THOMAS HOBBES*,

And now Translated into English

By *VEN. MANDEY.*

LONDON,

Printed Anno MDCLXXXI.

UMI

SOME Principles *and* Problems IN GEOMETRY, *&c.*

CHAP. I.

Of the Subject, Principles, and Method of the Mathematicks.

HE that would apply himself to the study of the Mathematicks, must in the first place understand what the matter is concerning which Mathematicians treat, and what it is that is enquired into about that matter.

We are therefore to know, that the matter which they treat of, is every thing that hath Magnitude, that is to say, all about which the question may be asked, *how great is it?* So that the subject matter of Mathematical Sciences, are the *Lengths*, *Superficies* and *Thickness*, or profundity of Bodies. For every Body hath three, and no more dimensions, and these differing in kind. From the motion of a Body, a thing long is produced, which is called a *Line*; then from the motion

motion of the Line ſprings a Superficie; and from the motion of the Superficie, *ſomewhat thick or groſs*, which is alſo called a *Solid and Mathematical Body*. There is no farther progreſs, for which way ſoever a Solid Body is moved, a Solid Magnitude will be deſcribed. They therefore, who among the kinds of Magnitudes rank Surdoſolids, Quadratoquadrats, *&c.* do idlely; for theſe are not Figures, but only Numbers, that is, not Magnitude, but Magnitudes.

Motion, Time, Force, Weight, belong likewiſe to the Mathematicks; for one of theſe may be ſaid to be bigger or leſs than another. Nay in ſo far the Mathematicks treat of qualities, as one may be called more or leſs ſuch than another. For of whatſoever may be Adverbially ſaid, more or leſs, that hath a foundation in ſome Magnitude. The Compariſon of Motions, Magnitudes, Times, *&c.* amongſt themſelves, pertains alſo to the Mathematicks; for by reaſon of the various *inequalities* of things, one *inequality* is more or leſs than another. *Inequality* therefore hath its own Magnitude, and that is it which is uſually called *Proportion* or *Reaſon*. Number belongs to the Mathematicks alſo; for to what Science a Body, or Motion, or Time, or a Line belongs in the Singular; all theſe belong to the ſame in the Plural. For when One is nothing elſe but a *thing numbred*, Number can be nothing, other than *things numbred*.

Now to ask how big a thing infinite is, would be an abſurd queſtion, and not at all to be anſwered. For neither can a finite Magnitude be divided into parts infinite in Number, nor can an infinite Number be given, when Mathematicians frequently ſay, *let a Line be drawn out in infinitum*, they are not ſo to be underſtood, as if they thought that would be done, but

that they leave it to be produced at the Readers pleaſure.

The thing that is enquired of thoſe ſubjects of the Mathematicks, is in a manner nothing elſe, than how great the propoſed Magnitude is, as it is compared with another Magnitude of the ſame kind; or that I may ſay it in a word, the Proportion is enquired. It is therefore neceſſary to one that ſtudies Mathematicks, exactly to underſtand theſe, and all other terms which Mathematicians uſe, that is to ſay, to know the true and clear Definitions of the ſame.

Quantity (which the Ancients no where defined, and who for want of a Definition thereof, raſhly diſputing about the nature of an Infinite, have erred) I thus define. Quantity is a Magnitude every way determined either by expoſition, ſo that to one asking how great is it, it may be anſwered as great as you ſee, or by compariſon; ſo that it may be anſwered, it is one or more feet great, or any other ſuch way. But above all things, he ought to have an accurate definition of Proportion or *Mathematical Reaſon*; and beſides, to know how to add, Subſtract, Multiply, and Divide Proportions, and the varieties of Compariſons, of which, though moſt of them be taught by *Euclid* and his Interpreters, yet I ſhall reduce almoſt all of them to the next Chapter.

The matter being known, and what we inquire into concerning it, the next thing is to take the Principles; the Principles of what? the Principles of the Science, or of knowing that we ſeek for: Now nothing but truths can be known. The Principles then of the Mathematicks, are the *firſt truths*, which we are not taught, but know them by the light of nature, ſo ſoon as we hear them ſpoken. They are therefore called the

the first Propositions, because of them the first of all Syllogisms consist. The Subject then of a Science, or any part of it, is not to be called a Principle; as a Point of Geometry, or an unity of Arithmetick, (which as a certain Master taught publickly) though the first Book of *Euclid*'s *Elements* begins with a Point, and the seventh Book with an unity.

The Principles of Mathematicks, or first Propositions, are Definitions; but that these may be useful, they must not only be true, but likewise accurate, lest that through so great variety of significations, we either proceed with uncertainty, or by continual distinctions being beat out of the Path, lose the truth by jangling. For as if a Hare should start amidst a company of Mastiffs, the Dogs let loose would fall upon one another, whil'st the Hare is running for it; so likewise in a Theatre of those who dispute for victory, truth flies away undiscerned. And for this cause (I mean to remove distinctions) Wise Men have introduced Definitions.

But to define well, that is, so to circumscribe with words the thing in hand, and that clearly, and with as much brevity as can be, that no ambiguity be left, is a very difficult task, and not so much a work of Art, as of the natural Intellect. However we are eased of most part of this difficulty by the Industry of the Ancient Geometricians; save that *Euclid*'s definition of a Line needs some correction, or at least a sound interpretation. It is true indeed, that in comparing of Longitudes, there is no respect at all to be had to Latitude; but yet in the construction of Figures, Latitude is necessary; for without Latitude, it is impossible to discribe a Figure; nor can it be known in what part of a broad Line that invisible Line is to be understood.

ſtood. The reſt though all of them be not moſt exact, yet becauſe they are true Propoſitions, they ſpoil not his Demonſtrations.

Now Definitions are of two kinds, of which the one barely explains the nature of the thing; and the other the cauſe or manner of producing it. But theſe Definitions are moſt uſeful to the advancing of knowledge, which contain the cauſes of, and manner of producing the thing defined. For that ſaying of *Ariſtotle's* is true, *To know is to know by the cauſe*; the reſt which only declare the eſſence of the thing defined, are generally leſs fruitful, for nothing follows from them that was not before contain'd in them; nor matters it, whether their Properties be called Definitions, or the Definitions Properties.

Petitions are uſually reckoned among Principles alſo; but theſe are not the Principles of knowing, but of conſtructing, that is, of the deſcription of Figures, wherein nothing is required, but that they be poſſible.

All Propoſitions likewiſe, whoſe truth is obvious to the light of nature, are to be held for Principles. Now there is no doubt to be made of the truth of a lawful Definition, becauſe they have their truth from the conſent and will of men, who at their pleaſure give names to things explained. But there are ſome Propoſitions, which though they depend on Definitions, and may by them be Demonſtrated, yet are ſo perſpicuous, that even without a Demonſtration they can force an aſſent. Theſe are called *Axioms*, but care is to be had, that they be not held for true upon the bare authority of Maſters. There is great difference between the authority even of the beſt Maſter, and the light of Nature. For there is nothing ſo abſurd, but that

ſome

some Master (in defence of some error) hath sometime written things as absurd.

The manner likewise of Adding, Subtracting, Multiplying, and dividing of Cossick Numbers, (that is, things numbred) is to be foreknown; which is easie, and taught by *Vieta* in his Isagoge, but most clearly and briefly by *Oughtred* in the Key of Arithmetick; from whence I have transferred as much as I thought fit into the third Chapter. The nature and manner of finding out Square and Cubick Numbers with their roots, is also to be learnt. This is handled in the fourth Chapter.

The nature of an Angle is likewise to be known; and therefore I have explained the properties thereof in the fifth Chapter.

To conclude, if he use any thing that is well Demonstrated by any Author for a Principle, he will do well; for every thing known stands instead of a Principle, for a farther progress in knowledge.

Some perhaps may say, that I place all Geometry in *præcognita, or things to be foreknown*; but they are mistaken. For he is a Geometrician (as I said before) who knows how to determine a Quantity by comparison amongst Quantities of the same kind. But those things which I would have foreknown, are only properties that attend Geometry, and steps whereby we are initiated in the Mysteries of sublimer Geometry, and whereof we stand in need not only for Figures and Mechanick works, but also for the knowledge of all natural causes, as being all contained in Motion.

The Principles and Theorems which are well Demonstrated in the Books of the Ancient Geometricians being well understood (for I write to those who de-

fire to improve the Science, the thanks for the work being due to the first Inventors, and no Man can reject what is already done, but who is very learned in his own conceit) the next thing is, (not to Demonstrate, but) to seek out the causes of Proportion that one Magnitude hath to another. For no Man can Demonstrate what he knoweth not, nor know but what he hath found out, nor (unless very seldom) find out what he hath not sought after. There is no certain Art of invention by Sagacity, Trying, Supposing, Deducing consequences from things supposed; Men often attain either to the causes of the matter sought after, or to something impossible. And by this Sagacity, (not by Art) all the Ancient Geometricians performed what they did, as manifestly appears, in that most of their Theorems begin with the Note of Supposition (*If*) But (one may say) how shall one chance to take that supposed which he stands most in need of? There is no certain method whereby the most convenient Suppositions may be chosen. And therefore we must try and experiment by measuring with a Compass and other Instruments, how near we can come Mechanically to the truth of the thing sought. For instance, in seeking the length of the circumference of a Circle, (all Men knowing by the light of nature, that any Arch is bigger than its Chord) it is an easie matter to come very near the true length. The remaining work is no more but to excogitate the possible causes of so great a Propinquity, wherein if they succeed, they have the principle of a Demonstration. For indeed in all doubts, nature it self sets us upon various Suppositions. But most Geometricians enquire nothing now adayes, but if it be *Euclid's*; though it be well known, that the most Noble Theorems

rems were invented, and the moſt beautiful Fabricks of the whole World built long before the time of *Euclid*. Now all our modern Geometricians almoſt, imagine to find not only the Proportions of Quantities, but alſo of Motions, that is, the cauſes of all natural effects, in the ſole extraction of the numeral roots, and that a falſe one too; and the reaſon is, becauſe ſo they are taught.

The laſt thing in Geometry is, the efficient cauſes of the thing ſought being found, to Demonſtrate from them as it were backwards. I ſay, from the efficient cauſes, becauſe the cauſes of numerical properties, are for moſt part nothing but arbitrary Impoſitions of numeral names; and Arithmetical Operations are the Demonſtrations of the Works themſelves.

CHAP. II.

Of Reaſon or Proportion.

1. REaſon is the *Relation* of a Magnitude to a Magnitude of the ſame kind, according to Quantity; as two Magnitudes, 2 and 5 being propoſed: The reaſon of the firſt to the ſecond is the relation of the Quantity of 2 to 5, that is, the Quantity of the Magnitude 2, as it is compared with the Magnitude 5. For the Inequality of two Magnitudes, may be more or leſs than the Inequality of two other things unequal.

2. Therefore for ſhewing Proportion or Reaſon, it is neceſſary that two abſolute Magnitudes be expoſed,

of which the former is usually called the Antecedent, and the second the Consequent.

So that neither both Quantities together, as 2 and 5, nor one alone, as 2 or 5; neither $\frac{2}{5}$ (therefore our Algebraists do err, who say that Fraction and Reason are the same thing) neither the difference betwixt 2 and 5, but the Quantity of Inequality that is betwixt two Magnitudes is that which is called Reason or Proportion. For though both 3 and 2, and 9 and 8, differ only by a unity, yet their Inequality is not the same. Yet if there be three Magnitudes, the least, the mean, and the greatest; the Reason or Proportion of the least to the mean, is greater than the Reason of the same to the greatest. But on the contrary, the Reason or Proportion of the greatest to the mean, is less than the Reason or Proportion of the same to the least. We might say that *Teucer* was bigger, compared to *Ulysses* than to *Ajax*; and *Ajax* was less, compared with *Ulysses* than with *Teucer*; the reason of this is manifest. For by how much a lesser Magnitude comes nearer a greater Magnitude, by so much it hath a greater Reason or Proportion to it, and by how much more a greater Magnitude exceeds a less, by so much, it hath also to it a greater Reason or Proportion.

3. Now the Proportions or Reasons of less to a greater, and of a greater to a less, are different kinds of Quantity; whereof the one is the Proportion of defect, the other of excess; of which, the former by how much the more it is multiplyed, by so much the defect becomes always the less, therefore it can never exceed or equal the Proportion of excess.

4. Reasons or Proportions are the same, or alike, or equal, in numbers, when they are of the same de-

no-

nomination, or may be reduced to the ſame denomination; as if both the firſt of the ſecond, and third of the fourth be $\frac{2}{3}$ or $\frac{4}{6}$: But where the Proportions are not expreſſible by numbers, both the firſt of the ſecond, and third of the fourth are correſpondent ſides of like Triangles; and are called Proportional, as having Portion for Portion by the Greeks Ἀναλογον, as the ſame thing ſaid again.

5. Continual Proportionals are ſuch as being Proportional have a middle Quantity common, as 1, 2, 4.

6. The root of a Square Number, is that Number which being multiplyed by it ſelf produceth ſome Number. Now the Number produced is called a Square or Quadrat, and the Root is always an aliquot part of its own Square; as if the Root be 8, the Square will be 64, and the Root the eight part of it.

7. It is then manifeſt, that the Root of a Square Number is not the ſide of a Square Figure, though ſome Algebraiſts deny this, being now aſhamed to confeſs the truth.

8. If there be three Magnitudes continually proportional, and from the ſame Antecedent three other continual Proportionals; the Proportion of the laſt to the laſt, will be the double of the Proportion of the ſecond to the ſecond; as is manifeſt in theſe numbers, 2. 4. 8; and 2. 6. 18, the Proportion of 18 to 8, is the double of the Proportion of 6 to 4. For as 18 is to 8, ſo is 9 to 4; but 9 to 4, hath a double Proportion of 6 to 4. Elem. 14 Prop 28.

9. Two Proportions are compounded or added together, when both Antecedents and Conſequents being multiplyed among themſelves, a Proportion is made: For example, if the Proportion of 2 to 3 be to be added to the Proportion of 4 to 5; let the Antece-

tecedents 2 and 4 be multiplyed one by another, and they make 8; then let the Conſequents 3 and 5 be multiplyed one by another, which make 15. This being done, the Proportion compounded of the Proportions of 2 to 3, and 4 to 5 is the Proportion of 8 to 15, according to *Eucl. Elem.* 6. def. 5.

10. Otherwiſe, if it be thus, as 4 is to 5, ſo 3 to another, which ſhall be $3\frac{1}{4}$, the Proportion of 2 to $3\frac{1}{4}$ (that is, both being multiplyed by four) will be of 8 to 15, that which is compounded of the Proportions of 2 to 3, and 4 to 5.

11. The Subtraction of a Proportion from a Proportion, is done by dividing the Quantities of the whole Proportion to be ſubſtracted by the Antecedent, and the Conſequent of the whole to be ſubtracted by both. For example, let the whole Proportion be of 8 to 15, the ſubtracted, of 4 to 5. I divide 8 and 15 by 4, the Quotients are 2 and $3\frac{1}{4}$. Again I divide the Conſequent of the whole Proportion 15 to be ſubſtracted by both the Quantities, to wit, 4 and 5, the Quotients will be $3\frac{1}{4}$ and 3, which are in the Proportion of 12 to 15, placing them in order 8, 12, 15. If from the Proportion of 8 to 15, the Proportion of 12 to 15, that is, of 4 to 5, be taken; it is manifeſt that there remains the Proportion of 8 to 12, that is, of 2 to 3.

12. Otherwiſe, if it be thus, as the conſequent of the Subtrahend, which is 5, is to its Antecedent 4; ſo the Conſequent of the whole, which is 15, is to the other 12; placing in order 8, 12, 15, it is plain that the Proportion of 12 to 15, that is, of 4 to 5, being ſubtracted from the whole Proportion of 8 to 15, there remains the Proportion of 8 to 12, that is, of 2 to 3.

13. Pro-

13. Proportion is multiplyed by saying, as the Antecedent is to the Consequent; so the Consequent to a third, and so the third to a fourth, *&c.*

14. A Proportion is divided by Number, when between the Antecedent and Consequent, one or more mean Proportionals are interposed.

15. If there be never so many Magnitudes of the same kind, the Proportion of the first to the last, is compounded of the Proportions of the first to the second, of the second to the third, and so forth unto the last; as in any Numbers, 4, 9, 10, 3, the Proportion of 4 to 3 is compounded of the Proportions of 4 to 9, and 9 to 10, and 10 to 3, which is Demonstrated by many.

16. But there is another kind of compared Quantity, which is also called Proportion, not Simply, but Arithmetical Proportion. Now four Quantities are said to be in Arithmetical Proportion, when the one constantly exceeds the other by equal differences. Concerning which Proportion, I shall only say, *If a Line were cut into as many equal parts, as it is intelligible it may be divided into, the Geometrical Proportional mean between every next two of these parts, will be the same with the Arithmetical mean.* If (for example) a given Line should be divided into the parts 2 and 16, the Arithmetical Proportional mean will be 9; so then 2, 9, 16 are continual Arithmetical Proportionals. Now the Geometrical Proportional mean is less than 6; but if the same Line had been divided into 100000 equal parts, and both a Geometrical and Arithmetical mean taken betwixt the whole and its parts 99999, how small a difference would be between those two Geometrical and Arithmetical Proportions? Therefore by how much less the difference be-

betwixt the extreams is, by so much less is always the difference between the Geometrical and Arithmetical mean. Wherefore if such means were every where interposed, all their difference would vanish. This is a most useful Proposition for finding out the Proportions that streight Lines have to crooked.

Chap. III.

Of Algebraical Operations.

1. The Algebraists instead of things, that is, instead of Magnitudes, put Letters as the Species of things, which in this manner we are taught to Add, Subtract, Multiply and Divide. If the Species be solitary, as A and B, they are added by interposing the Sign +, which signifies *more*. A and B then being added, the sum is A + B; but if B be to be subtracted from A, they interpose the Sign — which signifies less, or a mulct. Therefore A — B is that which remains of the Magnitude A; after that B is taken from it: Multiplying by a Number, is when the Number Multiplier is prefixed, as 2 A, 3 A, *&c.* The multiplication of A by it self is performed by writing A A or A q; a Line cannot be multiplyed by it self, and this is the first thing wherein the Algebraists of this Age differ from *Diophantus* and the Ancients, who contained themselves within the bounds of Arithmetick. If A be to be multiplied into B, they write A B, if it be written $\frac{A}{B}$, A is understood to be

divided

divided by B, which ſignifies the Quotient, as if A be 10, and B 2; that mark $\frac{A}{B}$ is 5, this does indeed very well in Numbers; but if A A be a Square Figure, and B the Diviſor a Line; then that Quotient $\frac{AA}{B}$ ſhews how often the Line B is in the Square Figure A A, which is againſt the nature of true Algebra, and likewiſe abſurd; for Planes or Flats are not compoſed of aggregated Lines.

But if the Species be affected with like Marks or Signs, as + A + A, they being added together make 2 A. For this is a true Rule of *Oughtred* and the Algebraiſts; in Addition the Species are to be added, and a common Mark to be affixed. If therefore — A be added to — A, the ſumm will be — 2 A; but if the Marks be different, then this is the Rule, let Species be ſubtracted from Species, and the contrary Mark prefixed to what remains; ſo that + A and — A added together make 0, that is nothing.

3. But if two affected Species be to be added to two affected Species, this is the Rule If they be like Signs, let the Species be added, reſerving the Signs. So A + B, and A + B added together make 2 A + 2 B. But if the Marks be different, as in theſe + A — B, and — A + B, the Rule is; let the like affected be reduced under their proper Signs, and the ſumm will be + A + B — A — B, that is nothing.

4. In Subſtraction, if the Species be ſolitary, and each have the Sign +, as + A + B, and + B be to be ſubtracted from + A, the remains is the Species from which the Subtraction is made, with the Sign +, together with the Species ſubtracted having the contrary

trary Sign. For B taken from A leaves A — B; but if both the Signs be —, the difference of the Species with the Sign + is to be taken for the remaining part, as if — 5 be to be subtracted from — 7, the remains will be + 2. For it is manifest, that — 5 is more than — 7 by two unities. And that it is so, is easie to be understood by any that will take the pains to consider it. For what is it else to take the defect from a given Magnitude, but to add to it the deficient Magnitude which it wanted? If therefore — A be to be taken from A, it is manifest that there remains 2 A. For by the very taking away of the defect, the accession of that which was wanting is added; in the same manner as he that remits a debt, bestows upon his Debtor as much as he hath remitted. If several Species affected with Signs be to be multiplyed by one or more Species affected, this is the Rule. If the Signs be like, let the Sign + be prefixed to the multiplied Species, but if different, the Sign — as in this example

A + B	A — B
A + C	A — C
+ CA + BC	— CA + BC
+ AA + AB	+ AA — AB
AA + CA + AB + BC	+ AA — CA — AB + BC

Which is true indeed in Numbers; but if they be Lines, false. For nothing can be multiplied but by a Number.

As many species as you please, and howsoever affected, are understood to be divided, when they place the

the Divisor under the Dividend, as $\frac{A+B}{B-C}$ is the Quotient of A + B divided by B — C, that likewise is true in Numbers; but if A + B be a Superfice, and B — C a Line, it is absurd, as we observed before. What remains is no more but labour in seeking out a commodious Supposition from whence they may begin, which is not art, but groping and chance. Now when they have found out any Proportion by Changing, Converting, Multiplying, Dividing, Compounding, and elevating and depressing it by a Scale of continual Proportionals, they toss the same, until that at length unawares, and by hap-hazard they stumble upon some Equation that may satisfy an Arithmetical Problem: But it is impossible by this method, to answer a Geometrical Question. Algebra is indeed, one of the Rules of Arithmetick, to the attaining the Theory whereof, two or three days at most are required, though to the promptitude of working, perhaps the practice of three months is necessary.

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CHAP. IV.

Of Square Figures, and Square Numbers.

1. A Square Figure and Square Number, when they have so many Square and equal parts, are one and the same thing; for a whole Figure, and the number of its parts, are the same.

2. If

2. If Square Figures be described from *crescent Radii* (or Radius's encreasing) as Numbers in natural order from an unity, they differ among themselves as odd Numbers of the same order. For the least Figure is one, to this the next odd Number (which is 3) being added, makes the next Square Figure (4). To this again the next odd Number (5) being added, makes the third Square (9) and so on. Therefore the excesses of the said Squares encrease as odd Numbers from an unity, to wit, 1, 3, 5, 7, *&c.*

3. If there be three Square Numbers next to one another in order, and a unity be taken from the middle, the number that is left is the mean Proportional betwixt the extreams. For example, 100, 81, 64 are Square Numbers next to one another in order. Let the *Unity* be taken from 81, there remains 80, which is a mean Proportional betwixt 100 and 64. In like manner 49, 36, 25, are Square Numbers next to one another in order. Take an Unity from the middle 36, there remains 35 the mean Proportional betwixt 49 and 25. So in these 9, 4, 1; if from the middle 4 one be taken, there remains 3 the mean betwixt the Squares 9 and 1. Lastly, 400, 361, 324, are Squares next to one another in order: If from the middle 361 an Unity be taken, there remains 360, the mean betwixt the extreams 400 and 324, and so in all.

4. Betwixt two next Square Figures, if to the least Square its Root be added, the whole will be the mean Proportional. For if to the Square 1, its Root (which is 1) be added, the whole 2 is the mean Proportional betwixt 1 and 4. In like manner, if to the Square 4 its Root be added, the whole 6 is the mean Proportional

tional betwixt 4 and 9; and so in all, as is manifest by induction.

5. If from a Square its Root be taken, there will remain the mean betwixt it and the next lesser Square. Let 100 be a Square Figure, if its Root 10 be taken from it, there will remain ninety, the Proportional mean betwixt 100 and 81, the next lesser Square. In like manner if from the Square 9, its Root 3 be taken, the remaining 6 will be the mean betwixt 9 and the next inferiour Square 4, and so always.

6. But if the Squares be not next to one another, but that one interposes, the less Square with two Roots is the mean Proportional betwixt those two Squares. For if to the Square 1 its two Roots, that is 2 be added, there will be 3, the Proportional mean betwixt 1 and 9. Likewise if to the Square 4 its two Roots, that is 4 be added, there will be 8, the mean betwixt 4 and 16; and so perpetually. The same mean will be had, if from the greater Square, its two Roots be subtracted. For if from 16, its two Roots be taken, there will remain 8, the mean betwixt 16 and 4.

If lastly two Squares have other two between them, and to the less Square the triple of its Root be added, you will have the mean betwixt the two extream Squares. For if 3 be added to the Square 1, there will be 4, the mean betwixt 1 and 16; or if to the Square 4 the triple of its Root, to wit, 6 be added, there will be 10 the Proportional mean betwixt 4 and 25; or if from 25 the triple of its Root, to wit, 15 be taken,

there will remain 10, the mean betwixt 25 and 4, and so always. From which follows a General Rule, that the mean Proportional betwixt two Squares, is so many Roots of the lesser added to the lesser; or what remains after so many Roots are taken from the greater, as the greater Square is distant places in the natural order of Numbers from the lesser.

7. Hence it is manifest, that the Root of a Square Number is not (as Algebraists imagine) the side of a Square Figure, nor at all a Line, because a Line addes nothing to a Flat, or Plane.

8. If two Squares are next to one another, both taken together, are greater than the double mean, by a Unity, or one square Particle. For 1 and 4 are next to one another, both together make 5, the double of the mean is 4, the difference is 1. In like manner 100 and 121 are next, both together are 221, the mean is 110, the double of the mean 220, the difference 1, and so in all.

9. If Squares admit of one between them, the extreams taken together are greater than the double mean by four. For 1 and 9 admit of one betwixt them, both together make 10, the double mean is 6, the difference 4. Likewise 100 and 144 receive one between them, the extreams put together are 244, the mean is 120, the double mean 240, the difference 4, and so every where.

10. If between two Squares, other two interpose, the extreams together are greater than the double mean by the number Nine. For 1 and 16 have

have two betwixt them, added together they make 17, the double mean is 8, the difference 9. Likewise 100 and 49 have two betwixt them, put together they make 149, the mean is 70, the double of that 140, the difference 9. In like manner, three being interposed, the extreams exceed the double mean by sixteen, and so forth by Square Numbers, throughout the whole Series of Squares.

11. Betwixt two Cubick Numbers, two mean Proportional Numbers intervene: Now they are found out after this manner; if the Cubes be next to one another, let the less Cube be divided by its Root, then let the Quotient be multiplyed by a Number that exceeds the Root by an Unity, the Product will be the less of the means. For example, take the next Cubes 1 and 8; let the lesser Cube 1 be divided by its Root 1, the Quotient will be 1, then multiply it by 1 more 1, that is 2. This number 2 is the less of the means betwixt 1 and 8; by the same method the greater mean is found out, 2 being divided by 1 the Quotient is 2, then 2 being multiplyed by 1 more 1, makes the greater mean 4. Lastly, 4 being divided by 1, the Quotient is 4, and 4 being multiplyed by 1 more 1 it makes 8; or let the less Cube be 64, its Root 4, the next greater Cube 125, 64 being divided by 4, the Quotient is 16; 16 being multiplyed by 4 more 1, makes 80, the first of the means. Again 80 being divided by 4, the Quotient is 20; which multiplyed by 4 more 1, makes 100 the greater mean, finally the last place if 100 be divided by 4, the Quotient will be 25, which being multiplyed by 4 more 1, makes 125.

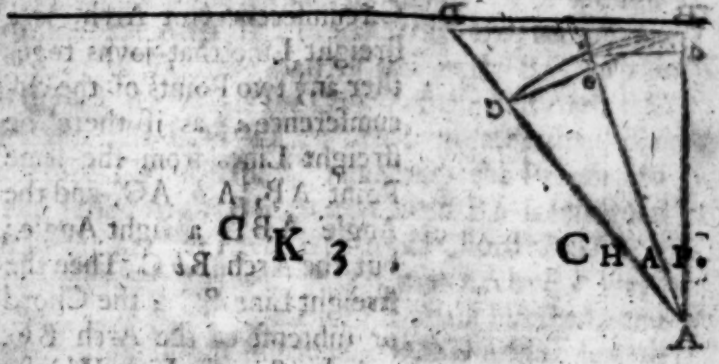
12. If two Cubick Numbers have one Cubick Number betwixt them, let the leſs Cube be divided by its Root, and let the Quotient be multiplyed by the Root more 2, the product will be the leſſer mean; let the Cubes be 1, 27: 1 being divided by 1, the Quotient is 1, which multiplyed by 1 more 2 makes 3 the leſſer mean. Again 3 divided by 1 is 3, then 3 multiplyed by 1 more 2, makes 9, the greater mean. Laſtly 9 divided by 1 is 9, which being multiplyed by 1 more 2 is 27.

13. In like manner if the Cubes be 8 and 64, 8 being divided by its Root 2, the Quotient is 4, and 4 multiplyed by 2 more 2 makes 16, the firſt mean, and 16 being divided by 2 makes 8, which multiplyed by 2 more 2, makes 32 the ſecond mean. To conclude, 32 being divided by 2, the Quotient is 16, which multiplyed by 2 more 2 makes 64. In like manner if the Cubes be 27 and 125, 27 being divided by 3, the Quotient is 9, which multiplyed by 3 more 2 makes 45 the firſt mean. Again 45 being divided by 3, the Quotient is 15, which multiplyed by 3 more 2 is 75, and 75 divided by 3, the Quotient is 25, and this multiplyed by 3 more 2 makes 125, and ſo always in Cubes that intermit one.

If two Cubick Numbers intermit two Cubick Numbers, the leſs being divided by its Root, the Quotient multiplyed by the Root more 3, will give the firſt mean. Let 27 and 216 be two Cubes intermiting two Cubes (to wit 64 and 125) 27 being divided by its Root, the Quotient is 9, which being multiplyed by the Root 3 more 3, makes 54, the firſt

first mean; 54 divided by 3 makes 18, which being multiplyed by 3 more 3 is 108. Lastly 108 being divided by 3, the Quotient will be 36, which multiplyed by 3 more 3 makes 216.

If two Cubick Numbers intermit three Cubick Numbers, and the less Cube be divided by its Root, and the Quotient multiplyed by that Root more 4, the product will be the first of the means. And so through all the Cubes of Numbers that are in a natural order, adding 5, 6, 7, &c. you may find the first means betwixt any two Cubes whatsoever.



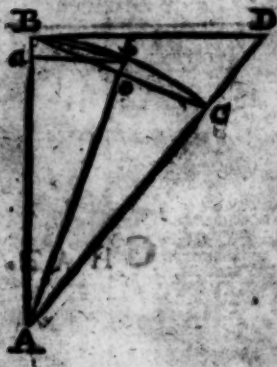
CHAP. V.

Of Angles.

1. A Streight or right Line Angle, is the digression of two ſtreight (or right Lines) from the ſame Point in the circumference.

2. The Quantity of a ſtreight Line Angle, is the Quantity of the Proportion that the intercepted circumference hath to the Perimeter of the Circle.

3. The Quantity then of an Angle is not a Line, neither a Superfice, nor yet a ſolid Body, but the Quantity of the Proportion which the Revolution of the *Radius* hath to another Revolution of the ſame.



4. The Chord of an Angle, Circumference, or Arch, is a ſtreight Line that joyns together any two Points of the circumference; as if there be ſtreight Lines from the ſame Point A B, A *b*, A C, and the Angle A B D a right Angle; but the Arch B *b* C. Then the ſtreight Line B C is the Chord or ſubtenſe of the Arch B C, and the ſtreight Line B *b* the Chord of the Arch B *b*.

5. The right Sine of the circumference is a ſtreight Line drawn from one term of the circumference, to the Radius which paſſeth through the other term perpendicularly; as B *e* or *a b*, is the right Sine of the Arch B *b*.

6. The

6. The Secant of an Arch or circumference, is a ſtreight Line drawn from the Centre by one term of the Arch to the ſtreight Line which is drawn perpendicularly in the other term to the Radius; as A D is the Secant of the Arch B C.

7. The Tangent of an Arch is that part which is intercepted of the right Line which is drawn perpendicularly from the beginning of the Arch; as B D is the Tangent of the Arch B C, or of the Angle A B C.

8. Hence it is manifeſt, that the right Sine of any Arch is the half of the Chord of the double Arch, or that the Chord of the Arch, is equal to the double right Sine of the half of the Arch, as if the Arch B *b* be the half of the Arch B C, *a b* doubled will be equal to the Chord B C.

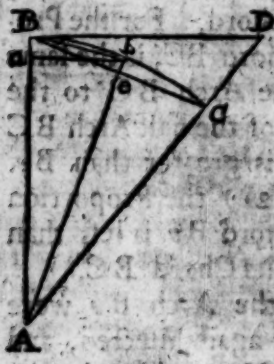
9. A leſſer Arch hath a leſs Proportion to its Chord, than a greater Arch hath to its Chord. For the Proportion of the Arch B C to its Chord B C, is the ſame with that of B *b*, the half of the Arch B C to the half Chord B *e*. But the Chord of the half Arch B C is the ſtreight Line B *b*, which is greater than B *e*. Therefore (by Chap. 1. Num. 2.) the Proportion of the leſſer Arch B *b* to its Chord B *b* is leſs than that of the greater Arch B C to its Chord B C.

10. Hence it follows, that if the Arch B *b* were biſected, and the Biſegment of it again biſected, and ſo on as often as can be done. We would at length come to a Segment of an Arch, the exceſs of which upon its own Chord ſhould be leſs than any given Longitude, and by conſequence leſs than the breadth of any Line having never ſo little Latitude.

11. In like manner, if a ſtreight Line were biſected, and the Biſegments thereof again biſected as often as might be, we would come at length to a Segment

leſs than any aſſignable Quantity, and therefore leſs than the breadth of a Line, having never ſo little Latitude.

12. Now that univerſal ſaying, *every Arch is greater than its Chord*, would hold univerſally true, if there could be given an Arch of a Circle, and a ſtreight Line, that had no breadth; but neither is there a Line without Latitude, nor if there were, or could be but ſuppoſed, could it enter into a Demonſtration, that handles the Proportion of Superficies and Solids; becauſe they are different kinds of Quantity, as appears by the fifth Definition, El. 5. of *Eucl.* Neither (as I conceive) would Dr. *Wallis* have thought other ways, but that that Definition is not found among the Principles of Algebra in *Oughtred*'s Key of Arithmetick.



13. In any Triangle, if the Angle be cut in two equal parts, and the ſtreight Line that cuts the Angle, cut alſo the Baſe; the Segments of the Baſe will be in Proportion of the Sides. This is clearly Demonſtrated in *Eucl.* Elem. 6. Prop. 3.

14. There is another kind of a plain Angle, which they call the Angle of Contact, ſuch as the Angle that the ſtreight Line BD makes with the Arch BC in the Point B. Which *Euclid* (or his Interpreter *Theon*) and many that have followed him, have affirmed to be greater indeed than any acute Angle; and the Angle which AB makes with the ſame Arch BC, bigger than any acute Angle, yet leſs than a right, which is manifeſtly abſurd. For then,

then, as Dr. *Wallis* will have it, the Angle of Contact had been of the ſame kind with the Angle that is made by the Revolution of the Radius; for every Angle that is not obtuſe, is either right, or leſs than a right Angle, by ſo much as any Angle is acute.

They were in the firſt place miſtaken in that they thought the ſpace betwixt the ſtreight Line BD, and the Arch BC to be an Angle, which (by Numb. 3. of this Chapter) is falſe. They were alſo miſtaken in that they took thoſe Angles for Quantities of the ſame kind, which is falſe. For an Angle of Contact, howſoever multiplyed can never equal nor exceed an Angle made by the Circulation of the Radius.

15. If it were true that the Angle made by the right Line AB with the Arch BC were leſs than a right Angle; it might come to paſs that an acute Angle encreaſing Uniformly, would at length become greater than a right, which yet could never equal it; the ground of which abſurdities is, that they thought no Line to have any Latitude, nor a Sine any Quantity at all.

16. That an Angle in the Centre, is the double of an Angle in the circumference, (if they inſiſt upon equal Arches) is Demonſtrated by *Euc.* El. 3. Prop. 20. But it is to be obſerved, that to the eſſence of an Angle in the circumference, it is not required that both the ſides ſhould terminate in the circumference; for DBC is an Angle in the circumference, though the Point D be not in the circumference.

CHAP.

CHAP. VI.

Of the Proportion of the Perimeter to the Radius of the Circle.

1. A Quadrantal Arch is equal to the Radius, together with the Tangent of 30 Degrees.

Let the Radius be AB, the half Radius BE, or AF, draw the Line then EF, the Rectangle ABEF will be the half of the Square from the Radius AB. By the Radius AB let the Arch BG be described, cutting EF in G: The Line then AG being drawn and produced to BE produced in H, BH will be the Tangent of 30 Degrees, and BG the third part of the Quadrantal Arch described by AB, then from the Point G to the side AB, let the Perpendicular GI be drawn. In BA produced, take AN the double of AI, so that IN be the Triple of IA, then draw the Secant NG, cutting AF in T, and produce it to BH in *q*.

I say that the streight Line B*q* is equal to the Arch BG.

Upon AT describe a Quadrantal Arch ST, because then the Triangles NIG, NAT are alike; AT will be two thirds of the streight Line GI the half Radius, that is, the third part of the whole Radius.

Because therefore AT is the third part of the Radius, the Arch ST will be equal to the Arch BG. Let BG be cut in two in the middle in *i*, and AT in M, and let the Perpendicular *ib* be drawn to AB,

and

and AM will be the fifth part of the Radius AB, and the Arch YM equal to the Arch B*i*; and as N*b* is to NA, so shall *bi* be to AM: Draw then and produce NM, it will pass through *i*; now let it be produced to BH in Q. Let *bi* be produced to the Concourse with N*q* in *t*, and *bt* will be the double of *bi*, and equal to the Chord of the Arch BG. For the right Sine of the half of any Arch, is the half of the Chord of the double Arch. Again let the Arch B*i* be bisected in *e*, and AM in L; and draw the Perpendicular *ea* to AB, and *ea* will be the half Chord of the Arch B*i*, and the fourth part of the two Chords of the Arches B*i* and *i*G. Now let AL also be the fourth part of the streight Line AT; draw then and produce the Line NL, and it will pass by *e*: Now let it be produced to BH in P. Therefore the Arch KL is equal to the Arch B*e*; in like manner if MT be divided in X, draw the Line NX and produce it, it will cut the Arch BG in *o*, so that *o*G is the fourth part of the Arch BG, and the Quadrantal Arch VX will be equal to the Arch B*o*, and so perpetually. The streight Lines then which are drawn from the Point N, will divide both the streight Lines AT, B*q*, and the Arch BG into the same Proportions.

If then the Bisection were continued as much as may be, we would at length come to the Arch of a Circle, less than the Latitude of the Line or side AB. Also according to Chap. 5. Num. 9. The Proportion of an Arch to the Chord always decreaseth. Wherefore the difference of the Arch to its own Chord decreaseth also, until it become less than any assignable difference.

Let us then suppose the Latitude of the Radius AB

to

to be equal to the least Segment, or less than it (for by bisecting, it can never be reduced to nothing) the last and least Segment will be in the streight Line BH to *q*, and so continually all the equal Segments of the Arch BG will lye extended towards the streight Line B*q*; for B*q* is greater than *bc*, which is equal to the Chord of the Arch BG. Let *ac* be produced till it meet with N*q* in *n*, and *an* will be equal to four Chords of the Arch B*c*, but less than the whole B*q*. In like manner, if the Arch *ac* be bisected, and the right Sine of it drawn, that produced till it meet the streight Line N*q*, it will be equal to eight Chords of the Arch *ac*, but less than the streight Line B*q*, and so always, until one come to the last and least Segment, whose Longitude is equal to the Latitude of the Radius AB.

Therefore if the least Latitude be allowed to the side AB, the Arch BG, and streight Line B*q* will be equal; but if all Latitude be denyed to a Line, not only the least part of an Arch will be greater than its Chord; but also all its parts to all the others, so that no Arch can be equal to a streight Line. Neither can there be any Line drawn, nor Sine, nor Arch, nor Figure. For a Line without Latitude is nothing, and the half of it as much.

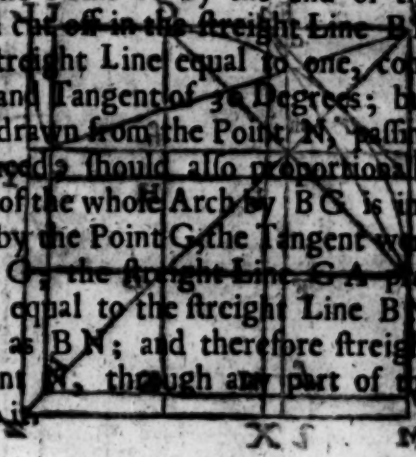
Therefore B*q* is equal to the Arch BG, that is, to the third part of the Quadrantal Arch described by the Radius AB.

Now because the Triangles GTF, E*Gq* are Rectangular and alike, and TF the third part of the half Radius AF; E*q* will be the third part of EH (the excess of the Tangent BH above the half Radius BE). The Triple then of B*q* (that is, the Quadrantal Arch described by the Radius AB) is equal

to

to the Triple of the Semiradius, together with the Triple of D q; that is, to the Radius A B, together with the Tangent of 30 Degrees, that is, together with B H, which was to be Demonstrated.

2. If B q were produced until it were Tripled, and A F until it were doubled, it is manifest that a streight Line drawn from the Point N by the end of the doubled A F would cut off in the streight Line B H being produced, a streight Line equal to one, composed of the Radius and Tangent of 30 Degrees; but that a streight Line drawn from the Point N, passing through A D produced, should also proportionally divide what remains of the whole Arch by B G, is impossible; because if by the Point G, the Tangent were drawn to the Point G, the streight Line G A produced until it were equal to the streight Line B N, would do the same as B N; and therefore streight Lines from the Point N, through any part of the Arch, would not do it.



SCHOLIUM.

3. Because I have shewed in Lib. 2, (of *Geometrical Rays*) that a Quadrantal Arch is equal to a right Line, whole Square is ten times as great as a Square from the half Radius; and have now made out, that the same Arch is equal to a streight Line, made up of the Radius and Tangent of 30 Degrees. In this place I will from most clear Principles demonstrate, that a Square from that streight Line composed of the Radius, and Tangent of 30 Degrees, is also the Decuple of a Square from the Semiradius.

De-

C
H
D
Q
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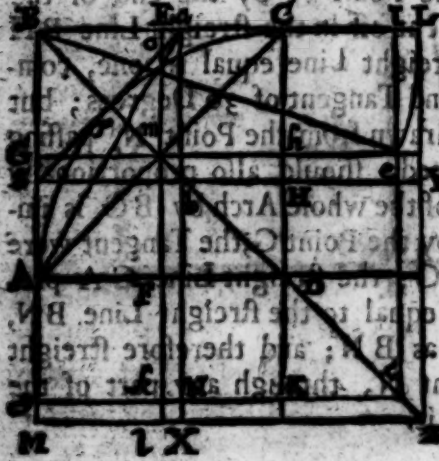
Describe a Square A B C D, and let it be divided four ways, as well by the right Lines E F, G H, as the Diagonals A C, B D concurring in the Centre of the Square at *m*; and describe the Arch A C cutting E F in *o*; and let it be produced to the side B C in *a*. B *a* is then the Tangent of 30 Degrees; let the side B G be produced, and in it being produced, let C I equal to the half side B E, and C L equal to the Tangent B *a* be taken. Then describe a Square from the whole B I, which is B L Z M, which is to be Demonstrated to be equal to Ten Squares from B E or C I.



The Square from B E is B E *n* G, the Square from the Tangent B *a*, is B *a b* G S, to which let C L Y H *b* be equal.

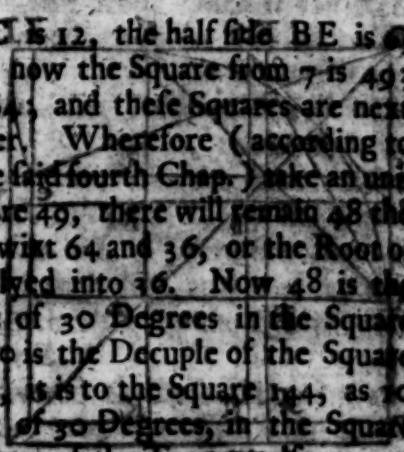
Describe also a Square from B I, which shall be B I *c g*, and let C D, E F be produced to the side M Z cutting *g c* in K and *k*. Let *a b* also be produced to the side M Z in X, and S H to L Z in Y.

Understand now that the four sides of the Square A B C D, are each divided into twelve equal parts. So it will be that the Square A B C D does contain 144 equal little Squares.

Because the side B C is 12, the side B I is 18, the Square of the former is 144, of this 324; the Square from

from 19 is 361, the Square from 20 is 400; and these square numbers are in order next to one another. Wherefore (according to Chap. 4 Num. 3) an unity being taken from the mean Square 361, the remaining 360 is the mean Proportional betwixt 400 and 324, that is, the Root of the square Number from 400 multiplyed into 324.

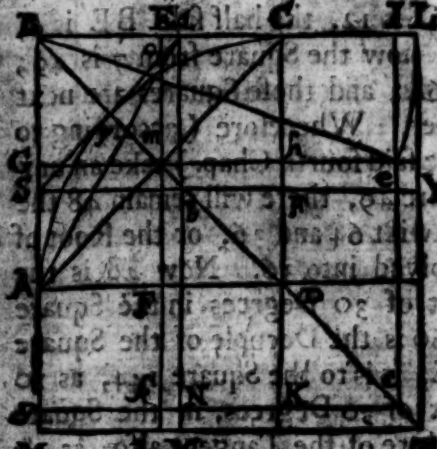
Again, because BC is 12, the half side BE is 6, which squared is 36; now the Square from 7 is 49; the Square from 8 is 64; and these Squares are next to one another in order. Wherefore (according to the same number of the said fourth Chap.) take an unity from the mean Square 49, there will remain 48 the mean Proportional betwixt 64 and 36, or the Root of the Number 64 multiplyed into 36. Now 48 is the Square of the Tangent of 30 Degrees in the Square 144; because then 360 is the Decuple of the Square from the half side BE, it is to the Square 144, as 10 to 4; so the Tangent of 30 Degrees, in the Square 360 will be to the Square of the Tangent also, as 10 to 4; but the Tangent of 30 Degrees in the Square 360, to wit, the third part of it is 120, which number is to 48, the Square of the Tangent in the Square 144, as 10 to 4. Therefore 360 is the Square from the right Line composed of the Radius and Tangent of 30 Degrees, which right compounded Line is BI, which was to be Demonstrated.



4. In the right Line Ie, take Ie equal to the streight Line CI, which being joyned to BE, the Arch of the Circle described by the Radius Be will pass by L; for BI hath the power of nine Squares, whereof Ie hath the power of one; therefore Be hath the power of ten half sides, the Arch therefore described by it will pass by L.

5. We

5. We must not here pass over a difficulty, which in the 23 Chap. of the Book of Principles; I did indeed mark, but thought fit to leave the Solution of it to the Readers: The Solution of this will much conduce to the illustrating and facilitating of Geometry. This is the difficulty, the Square CLYH (which is the Square of the Tangent of 30 Degrees, and equal by Construction to B*b*S) is to the Square C*l*, as 4 to 3, as is commonly known. Because therefore the Gnomon IL Z*c*M, ought to be equal to the triple of the Gnomon IL Y*c*H, and so that Gnomon being tripled and placed upon this ought to agree with it, but it agreeth not, for it is less. Hence it may seem conclusive, that the side of the Square which is the Decuple of the Square from the half side, is greater than the Square from BI; but why? because it is less by so much, as the double of the Square *mb*; or *c*Z or *c*Y.



For suppose the right line BC to be divided into twelve equal parts, the whole Square ABCD will be 144 of those parts; the Square CIYH 48, the Square also AX 48; and the Square DZ 48, all these make 288. Now seeing the Square from the half side CI is 36, the whole Square (which is ten

times

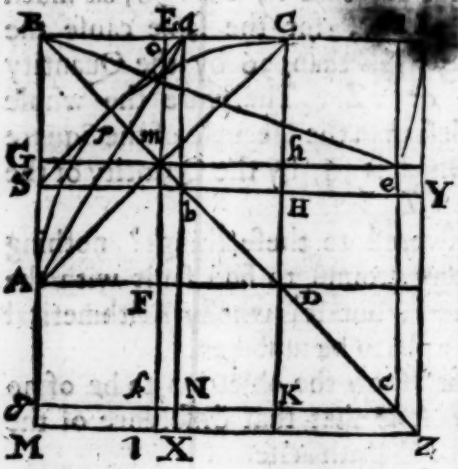
times as much) ought to be 360, 288 being then taken from 360, the remainder 72 ought to be equal to the two equal Rectangles X D and D Y, and either of them ought to be 36, but it is not so. The Square F D K *k* is indeed equal to the Square from B E or C I (that is 36) from which if the Rectangle F *l* be taken, and the Rectangle X K added (seeing that is greater than this by the Quantity of the Square *k* X) the remaining Rectangle X D will be less than the Square F K, that is, than 36, as much as the Square *k* X [illegible], for the same cause the Rectangle D Y is also less than 36 by the Quantity of the Square *e* Y or *c* Z. Therefore the whole Square B L Z M is less than the Decuple of the Square from the half-side, which is 36, by the Quantity of the double little Square *c* Z.

What can be answered to these things? nothing truly, if there be any ground to find fault with the former Demonstration; but it is wholly Arithmetical wherein it is not so easie to be mistaken.

I will therefore first shew the objection to be of no force, then whence it is that that difference of the double little Square *c* Z doth arise.

That it is of no force, is manifest from hence, that by the same argument it may be proved (it being supposed that the right Line B C is divided into twelve equal parts) that the Square of it also is less than 144, by the Quantity of the same double Square *c* Z. For seeing the Square C *m* is 36, the right Angle C *a b* H will be less than it by the Quantity of the Square *b m* or *c* Z. For from the Square C *m* on the one side, the complement E *b* is taken, on the other *b h*; but E *b* is greater than *b h* by the Square *b m*. Therefore the Rectangle C *a b* H is less than 36, by the same

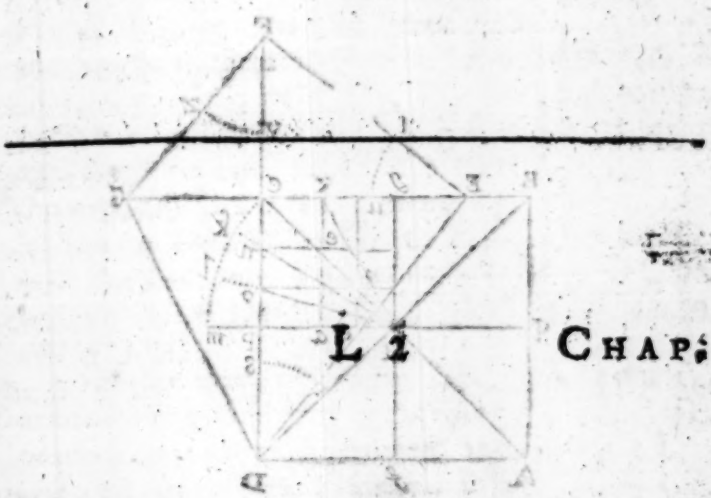
little Square cZ; for the ſame reaſon the Rectangle Ab is leſs than 36, by the ſame little Square cz; there remains for compleating the whole Square, the Square bD, which is the half of the Square from the Tangent Ba. For joyn as cutting the Diagonal in p, it will be as Sa, or Bb to Ba, ſo Ba to bp, that is, to Hb. ab is then the mean Proportional betwixt Bb and bH; therefore the Square bD is the half of the Square $Babs$, that is 24; therefore the whole Square 144 is equal to 48 + 24 + 36 + 36, leſs the double Square cZ, which is abſurd. For it is not to be doubted, but that the Square ABCD is 144. Therefore this objection is of no force, and has this fault beſides, that the Square from BL is by it made leſs than 360 not by the double, but the Quadruple of the Square cZ.



The cauſe of this diſagreement is therefore to be enquired into, and it can be no other, but the reckoning of Lines without Latitude in the Proportions of Superficies, whence it follows neceſſarily, that that which is contained within the four ſides BL, LZ, ZM, MB is leſs than ten Squares from BE, by the double of cZ; but that the ſides themſelves are equal to the Quadruple of cZ. Therefore that the Square from

BL

BL is the Decuple of the Square from the half ſide, ſtands good and ſhall remain ſo; for it cannot be proved by any argument, that the ſquare from BL is leſs than the Decupled Square from BE, but that by the ſame it will be proved that the Square from BC is leſs than four Squares of the ſame BE. We have then by the Solution of this difficulty removed one, and perhaps the greateſt hinderance to Geometry, to wit, the computation of Superficies by Lines without Latitude.

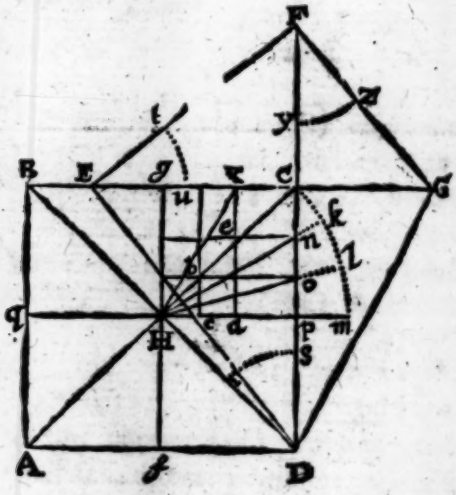


CHAP. VII.

Of mean Proportionals.

1. HOw to find a mean Proportional betwixt two right Lines given, is taught by *Euclid's El.* 6. Prop. 13.

2. To find out two mean Proportionals betwixt a right Line given and its half; let DC be the right Line given, and its half CG; let them be disposed



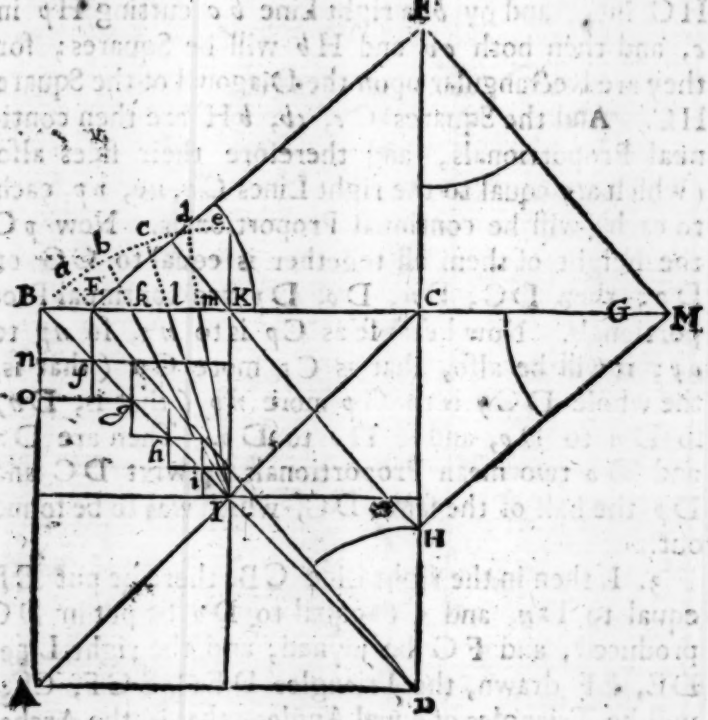
to right Angles in C; then make a Square DABC, and let it be cut as well by the Diagonals AC, BD, as by the right Lines *gf*, *qp*, meeting four ways in H. Take the mean Proportional betwixt BC and its half

half CG; now that is equal to the Semidiagonal HC, by the Radius HC describe the Arch C*m*, cutting *q p* produced to *m*, and let the Arch C*m* be cut into three equal parts in *k* and *i*, and draw H*k*, H*i* cutting DC in *n* and *o*. Now let C*x* equal to C*n* be taken; draw *x a* parallel and equal to C*p* cutting HC in *e*, C*e* then is a Square. In like manner by the point *o*, draw a parallel to the side BC, cutting HC in *b*, and by *b* a right Line *b c* cutting H*p* in *c*, and then both *eb* and H*b* will be Squares; for they are Rectangular upon the Diagonal of the Square HC. And the Squares C*e*, *eb*, *b*H are then continual Proportionals, and therefore their sides also (which are equal to the right Lines C*n*, *no*, *op*, each to each) will be continual Proportionals. Now *p*C the height of them all together is equal to CG or D*p*; then DC, D*n*, D*o*, D*p* are continual Proportionals. Now because as C*p* is to *np*, so *np* to *op*; it will be also, that as C*p* more C*p* (that is, the whole DC) is to C*p* more *np* (that is, D*n*) so D*n* to D*o*, and so D*o* to D*p*. Then are D*n* and D*o* two mean Proportionals betwixt DC and D*p* the half of the same DC, which was to be found out.

3. If then in the right Line CB, there be put CE equal to D*n*, and CF equal to D*o* be put in DC produced, and FG be joyned, and the right Lines DE, EF drawn, the Triangles DEC, ECF, CFG will be Triangles of equal Angles, that is, the Arches *sr*, *tv*, *zy*, (having equal shanks) will also be equal.

4. How to find four mean Proportionals betwixt a right Line given and its half.

Let the right Line given (in the second Figure) be DC, and the half of it CG; make a Square DCBA, the fourth part of which is the Square IB. So then the streight Line IB is the mean Proportional betwixt the whole BC and its half CG.



By the Radius IB describe the Arch B*e*, cutting IK produced to *e*; and let it be cut into five equal parts by the streight Lines I*a*, I*b*, I*c*, I*d* cutting BK in E, *k*, *l*, *m*, and let B*n* equal to BE be taken.

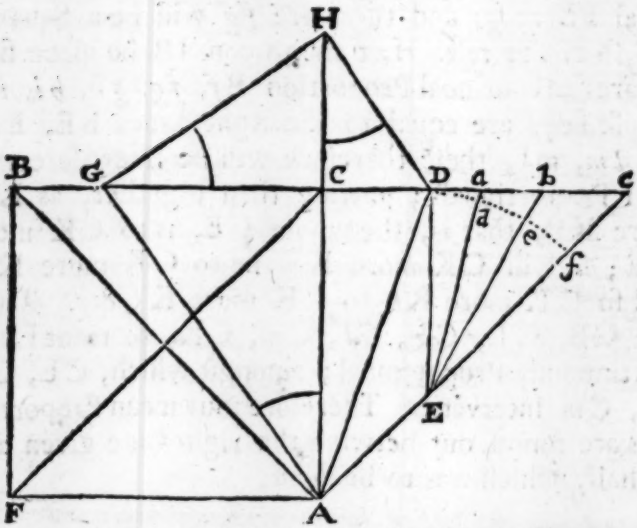
The

The Lines *nf*, E*f* being then drawn, the one parallel to BE and the other to B*n*, they will meet in the Diagonal Line IB at *f*, and B*f* will be a Square. In the ſame manner, if B*n* be lengthened to *o*, that it be equal to B*k*; right Lines drawn by *o* and *k* parallel to the oppoſite ſides, will meet in the Diagonal IB at *g*, and therefore *fg* will be a Square, and ſo of the reſt. Let then upon IB be made five Squares in continual Proportion B*f*, *fg*, *gh*, *hi*, *i*I, whoſe ſides are equal to the right Lines BE, E*k*, *kl*, *lm*, *m*I, theſe therefore will be likewiſe continual Proportionals; putting then together, as KB more KB, that is, the whole CB, is to CK more KE; ſo will CK more KE be to CK more K*k*, and ſo CK more K*k* to CK more K*l*, &c. Then are CB, CE, C*k*, C*l*, C*m*, CK, ſix right Lines continuouſly Proportional; amongſt which, CE, C*k*, C*l*, C*m* intervene. Therefore four mean Proportionals are found out betwixt the right Line given and its half, which was to be done.

5. Conſectary, If in the right Line DC produced, CF equal to C*k* be put; and CM equal to C*l* be put in BC produced, and in CD there be put CH equal to C*m*. And laſtly, if HK, HM, MF, FE, ED be joyned, there will be made five Equiangled Triangles, DEC, CEF, FMC, MHC, HKC.

6. How to find two mean Proportionals between two ſtreight Lines whatſoever given.

Let (in this 3 Fig.) BC the greater and CD the leſſer be given. Let them be diſpoſed at right Angles in C, and let BC be equal to AC, let AD, AB be joyned, and C*c* equal to the right Line BC be taken, and A*c* joyned.



From the Point D, draw down DE, making right Angles at D, and cutting A*c* in E: From the Centre E at the diſtance ED, deſcribe an Arch of a Circle D*f*, ending in A*c* at *f*; let the Arch D*f* be cut into three parts in *d* and *e*, and the Lines E*d*, E*c* being drawn, produce them to D*c* in *a* and *b*. According then to the 2 art. of this Chap. *c b*, *b a*, *a* D, will be continual Proportionals; and (by El.6.Prop.1.) the Triangles A CD, A D*c*, will be as the Baſes CD, D*c*; and the Triangles A C*c*, E D*c* alike; and the Triangle A C*c*, will be to the Triangle E D*c* in the double Proportion of C*c* to D*c*; therefore C*c*, C*b*, C*a*, CD, will be continual Proportionals.

7. Con-

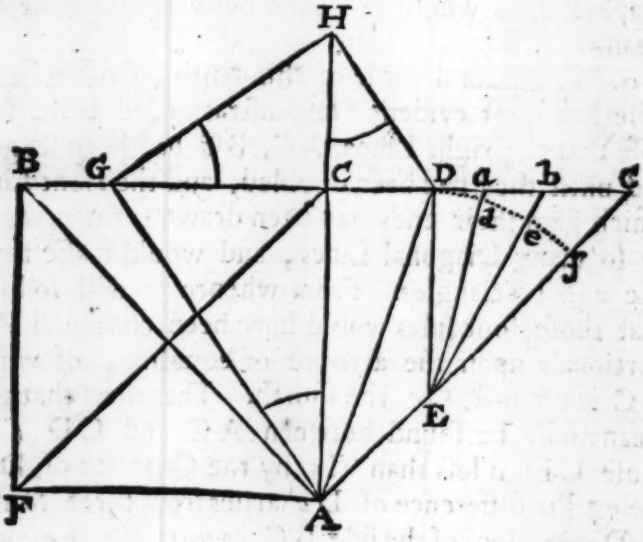
7. Conſectary, If then in the right Line CB, there be put CG equal to C*b*, and in AC produced CH equal to C*a* be put, HD being drawn, and AG, GH joyned, the Triangles ACG, GCH, HCD, will have equal Angles.

8. In like manner if CD, were put in the ſide CB, and the Arch D*f* cut in five parts, between AC and CD, four mean Proportionals would be found; but if CD were put in AC produced, and the Angle DE*c* cut in ſeven parts, ſix mean Proportionals would be found betwixt the ſame extreams.

9. The natural cauſe of this truth, from whence ariſeth a moſt evident Demonſtration, is this; ſuppoſe that the right Lines AC, BC had been lengthned until they had been doubled, and the right Lines which joyn their ends had been drawn; theſe would be ſo many Diagonal Lines, and would make three like equal Triangles. From whence it will follow, that thoſe four ſides would have been continual Proportionals upon the account of equality, of which AC is the firſt, C*c* the fourth. Therefore that two means may be found between AC and CD (becauſe CD is leſs than C*c* by the Quantity of D*c*) ſeeing the difference of D*c* ariſes from three Mulcts or Diminutions of the ſide BC, to wit, *c b*, *b a*, *a* D, it is neceſſary that the Difference D*c* be divided into three parts *c b*, *b a*, *a* D continual Proportionals. and becauſe the Angle CAB is half a right Angle, the Angle alſo DE*c* (from which Point E the diviſion of the right Line D*c* into three parts, being continual Proportionals taketh its riſe) ought likewiſe to be half a right Angle.

Now

Now seeing that that Division into three, is done by a Trisection of a half right Angle, and no otherwise, and in taking four means by a Quinquisection or fivefold cutting, and no otherwise. Also in taking six Proportional means, by Septisection or a sevenfold cutting, and no otherwise, and so forward: No other method will serve to find out means that are even in number, as 2, 4, 6, 8, *&c.* except the Trisection, Quinquisection, Septisection, *&c.* of a half right Angle.



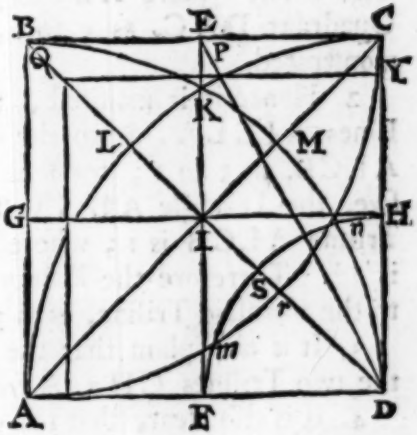
CHAP.

CHAP. VIII.

Of the Proportion of a Square, to the Quadrant of a Circle inſcribed in it.

1. LEt the Square ABCD be divided not only by the right Lines EF, GH, but alſo by the Diagonals AC, BD meeting fourfoldly at I in the Centre of the Square, and cutting the Arches AC, BD in L and M; and deſcribe from the Centres C and D, the Quadrantal Arches AC, BD mutually cutting one another in the right Line EF at K.

I ſay that the Square ABCD is to the Quadrant DAC, as 5 to 4.



Though I have demonſtrated this in another place, yet for the ſake of thoſe, who cannot ſo eaſily go along in their thoughts, with long and difficult demonſtrations, I ſhall in this place demonſtrate the ſame by a ſhorter and eaſier method. By the Radius DH, deſcribe the Arch of a Quadrant HF, cutting the Arch BAC in *m* and *n*, and the Diagonal BD in *s* and *r*; the Quadrant

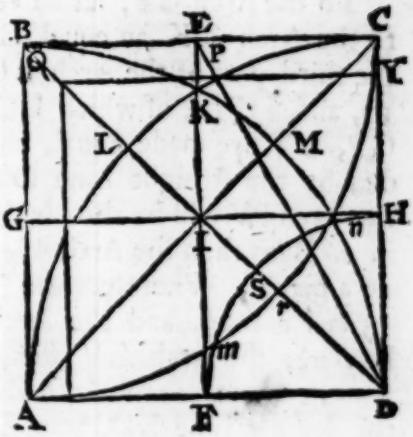
drant then DHF, is the fourth part of the Quadrant DCA, and the ſpace CLF*s*H, is three fourths of the ſame. Now the ſame ſpace CLF*s*H, is three fourths of the Quadrant BA*r*C; taking then that common ſpace CLF*s*H from both the Quadrants, there will remain on the one part the Quadrant DHF, on the other part the Triline or three Lines DA*r*C leſs than the Biline or two Lines *m n*, more than the two three Lines CH*n*, AF*m*, that is, the ſame three Lines DA*r*C, and the Quadrant DHF equal to one another; and therefore the ſpace BAF*s*HC, will be equal to the Quadrant BA*r*C. For that ſpace conſiſts of the Triple of the Square DI, and the fourth part of the three Lines ALCB, that is, of three fourths of the Quadrant, and four fourths of the Triline ALCB. Seeing then the three Lines ALCB, are equal to the fourth part of a Quadrant: The whole Square ABCD, will be to the whole Quadrant DAC, as 5 to 4, which was to be Demonſtrated.

2 Hence it is manifeſt, that the Biline or two Lines AICLA, is to the Triline or three Lines ALCB, as 3 to 2; for ſeeing the whole Square is five, the Triangle ABC will be $2\frac{1}{2}$, of which the Triline ALCB is 1; wherefore the Biline AICLA is $1\frac{1}{2}$. Therefore the Proportion of the ſaid Biline to the aforeſaid Triline, is as 3 to 2.

3. It is alſo plain that the Biline *m n*, is equal to the two Trilines CH*n*, AF*m*.

4. It is alſo clear, that if a right Line were drawn parallel to the ſide BC by the Point P, in which the ſtreight Line DE cuts the Arch CL, and ends in the ſide DC at Y, and in the Diagonal BD at Q. The Square from YQ would be equal to the Quadrant

drant DAC; seeing the Square ABCD to the Square from YQ is as 5 to 4; and the Trilines CYP, PQL equal.



5. It is also manifest, that the half of the Square ABCD, to wit, the Rectangle AE, the Figure AKF being taken away, is the fifth part of the Square ABCD. For that which is contained within the three Lines ALK, KB, and BA, together with the Triline BEK, is equal to the Triline ALCB, that is, the fifth part of the Square ABCD; because BEK, CEK are equal, the same also is true of the Rectangle DE. Whence it follows, that the two Figures FKD, FKA taken together, are equal to three fifths of the Square ABCD, that is, to three fourths of the Quadrant DAC.

Now seeing the knowledge of these things is of it self of no great use; I might have passed them over, but that as I had begun to handle Cyclometrie, I was willing also to perfect it. Likewise because to the fulness of Cyclometry, it belongs to know also the Quantities of its parts, that is, of its Segments or Angles; we must now treat of these.

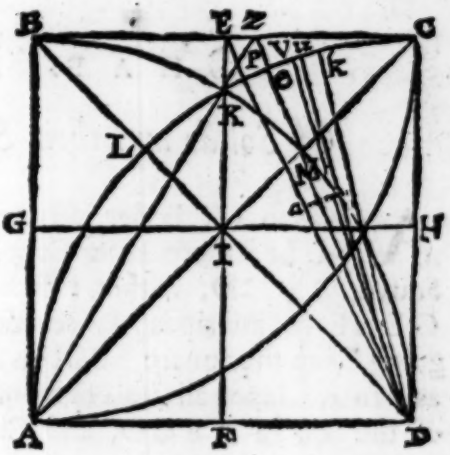
6. Describe again (in the second Figure) a Square ABCD, divided as in the first Figure.

To

To the Arch CP, let an equal LV be taken, and to the Arch LK an equal C*k*; becauſe then LK is the third part of the Arch LC; LK will be 10, K*k* 10, and *k*C 10, of which LC is 30. Again becauſe CP, LV are made equal, the Arch CL being divided by the ſtreight Line D*e* in two in the middle, the Arch PV will be alſo divided in two, in the middle in *e*. Both then the Arch C*e*, and the Arch L*e* are 15 a piece. Wherefore the Arch V*e*, as well as the Arch P*e* is $2\frac{1}{2}$, and the Arch PV 5, and the Arch KV $7\frac{1}{2}$, the Arch LV then as well as the Arch PC is $17\frac{1}{2}$; therefore the Arch CV or KP is $12\frac{1}{2}$. From the ſtreight Line ED, take E*a* equal to the half ſide EC, and the remainder D*a* (according to the 13 *E.* Prop. 1) will be the greater Segment of the ſide DC, or of the ſtreight Line DP divided in extream and mean Proportion. At the diſtance D*a*, deſcribe an Arch *ab* cutting the Arch DB in *b*; and let D*b* be produced to the Arch CL in *v*; the Arch C*v* then will be 12, of which CL is 30. For (by El. 14. Prop. 9) the ſtreight Line D*b* ſubtends the tenth part of the whole Perimeter, that is, a fifth part of the Semiperimeter, that is, two fifths of the Arch DB. The Arch then D*b* is two fifths of the Arch BD; and becauſe the Angle in the Centre, to wit, DA*b* is the double of the Angle in the circumference, to wit, of the Angle CD*v*, the Arch C*v* will be two fifths of the Arch CL. Therefore ſeeing CL is 30, C*v* will be 12; now becauſe the Arches LV, CP are equal, CV will be $12\frac{1}{2}$, of which LC is 30. Wherefore as well LV as CP ſhall be $17\frac{1}{2}$, and P*e* or *e*V 2_2, and PV or K*e* 5.

If we ſuppoſe the Arch CL to be 45, yet the Proportion of the Angles will be found to be the ſame.

For



For L K, K *k*, *k* C, will be each of them 15, and L *e*, *e* C each $22\frac{1}{2}$, and K *e* $7\frac{1}{2}$, L P $18\frac{1}{4}$; wherefore C P will be $26\frac{1}{4}$, and L V as much, and C V $18\frac{1}{4}$. Now $18\frac{1}{4}$ more $26\frac{1}{4}$ make 45; but the ſame is the Proportion of 10 to 5, as of 15 to $7\frac{1}{2}$. It ſtands then good, that the Angle E D C, to the half right Angle L D C, is as $17\frac{1}{4}$ to 30, and to L P as $17\frac{1}{2}$ to $12\frac{1}{2}$, or as 7 to 5, and to L K (which is 10) as 7 to 4. We have then the Quantity of the Angle E D C that was not known before.

CHAP.

CHAP. IX.

Of Solids and their Superficies.

A Cube to a Cylinder inſcribed in it, is as 5 to 4.

1. Let there be a Cube, whoſe Baſe is the Square A B C D, within this Square let the Circle G E H F be inſcribed. According then to the foregoing Chap. the Square A B C D is to the Circle G E H F, as 5 to 4. Upon all the Points of the Compaſs, both of the Square A B C D, and Circle G E H F; ſuppoſe Perpendicular Lines erected, the height of every one of which, is equal to the height of E F. So a Cube will be deſcribed, together with a Cylinder inſcribed in it, and the Baſe of the Cube will be to the Baſe of the Cylinder, as 5 to 4. Now the Plane that cuts the Cylinder and Cube Parallelways to the Baſe, will every where make a Section of the Square, to the Section of the inſcribed Circle, as 5 to 4. Therefore as the Square A B C D to the Circle G E H F (that is as 5 to 4) ſo (according to El. 6. Prop 1.) all the Sections of the Cube together, (that is the Cube) will be to all the Sections of the Cylinder together (that is, to the Cylinder) as 5 to 4.

2. The Superficie of a Cube, is to the Superficie of a Cylinder inſcribed in it, as 5 to 4: For the Superficie of

of a Cube is equal to ſix Squares from the ſide or height of the Cube. Now *Archimedes* in his Book of a Sphere and Cylinder, hath Demonſtrated that the Superficie of a Cylinder without the Baſes, is equal to the four greateſt Circles in a Sphere, and with the Baſes, to ſix Circles. The whole Superficies then of a Cylinder, is equal to the ſix greateſt Circles in a Sphere. Therefore the Superficie of a Cube, to the whole Superficie of a Cylinder, is as ſix Squares to ſix greateſt Circles, that is, as one Square to one Circle inſcribed in it, that is, as 5 to 4.

3. A Cube to a Sphere inſcribed in it, is as 15 to 8. For a Cube to a Cylinder (as is already ſhewed) is as 5 to 4. Now a Cylinder to a Sphere (as *Archimedes* hath Demonſtrated in the firſt Book of a Sphere and Cylinder) is as 3 to 2. Therefore the Proportion of a Cube to a Sphere, is made up of the Proportions of 5 to 4, and 3 to 2; be it then as 3 to 2, ſo 4 to another; now that ſhall be $2\frac{2}{3}$. The Proportion then of 5 to $2\frac{2}{3}$, is made up of the Proportions of a Cube to a Cylinder, and of a Cylinder to a Sphere. Therefore a Cube to a Sphere is as 5 to $2\frac{2}{3}$, that is, (both being multiplyed by three) as 15 to 8.

4. A Sphere is equal to the half Cylinder in which it is inſcribed, together with the half of the Cone which is inſcribed in the ſame Cylinder. For ſeeing it is Demonſtrated by *Archimedes*, that a Cylinder is to a Sphere inſcribed in it, as 3 to 2, and to the inſcribed Cone, as 3 to 1, a Cylinder, Sphere, and Cone will be as 3, 2 and 1. But 2 (that is, a Sphere) is equal to the half of the Aggregate of 3 and 1, (that is, to the half of the Cylinder, and half of the Cone put together) as is propoſed.

5. *Archimedes* alſo in his ſecond Book of the Sphere

and Cylinder, Prop. 1, ſhews how a Sphere equal to a Cylinder may be found, to wit, that a Cylinder be taken, which to the Cylinder propoſed is as 3 to 2, that is, as the propoſed Cylinder to a Sphere, then that two mean Proportionals be found betwixt the height of the taken Cylinder, and the Diameter of the propoſed Sphere: But to find out two mean Proportionals betwixt two right Lines, was not then found out. How that is to be done, I have Demonſtrated in the ſeventh Chapter of this Treatiſe.

6. The Superficie of a Sphere, is equal to the four greateſt Circles in the ſame Sphere.

For if the Arch of the Semicircle E F be turned round upon the *Axis* E F, it will be the Superficies of the Sphere. From the Point F, let the ſtreight Line F I be how you will drawn to the circumference, and from the Point I draw the right Line I K parallel to the ſide B C, cutting the circumference in K, and the Axis in L; the Angles at L will be right; and as F L to L I, ſo will L I be to L E, and as F I is to F L, ſo will E I be to E L, becauſe of the like Triangles F L I, E L I. Now whil'ſt the Semicircle turns upon the Axis E F, let the Circle I K be deſcribed from the Point I in the Superfice of the Sphere; and it will ſo fall out in whatſoever Point of the circumference E F, the Point I be placed, but as F E is to L E, ſo is the Circle upon E F to the Circle upon L E; for Circles are in the double Proportion of Rays or Radius's.

Becauſe

Becauſe then all the Perimeters IK make the Superficie of the Sphere, and all the right Lines IK make the Circle GEHF; the Superficie of the Sphere will be to the Circle GEHF in the double Proportion of the Axis EF to the Radius of the Circle GEHF the half of the ſame EF. But the Circle deſcribed on the Axis EF is alſo to the Circle GEHF in the double Proportion of the Axis EF, to the Semidiameter of the Circle GEHF. Therefore the Superficie of the Sphere is equal to the Circle, whoſe Semidiameter is the Axis EF. Now the Circle whoſe Radius is the Axis of the Sphere, is the Quadruple of the Circle GEHF. Wherefore the Superficie of the Sphere is alſo the Quadruple of the ſame.

7. Hence it follows, that the Superficie of any Segment or Portion of a Sphere, is to the Superficie of the remaining Portion, as the Portion of the Axis cut off by IK is to the remaining Portion, to wit, the Superficie of the Portion FIK, is to the Superficie of the Portion EIK, as the Portion of the Axis FL, to the remaining Portion EL.

8. Laſtly, Becauſe *Archimedes* hath Demonſtrated, that the Convex Superficie of a Cylinder, is the Quadruple of the Circle, which is the Baſe of the Cylinder, it follows, that it is the ſame thing, as to the quantity of a Cylinderical Superficie, whether it be made by the Revolution of the Perimeter of the Baſis about the Axis, or by the motion of the Baſe by the ſides of the Cylinder, or by the circular motion of the ſide. For all theſe beget equal Quantities of Superficie; neither is it hard to be underſtood even without a Demonſtration, that there is no difference in making a Superficie, whether the Circle that is the Baſe of the Cylinder be carried ſtreight by the ſide, or whether in every Point

of the ſide Perimeters be ſeverally deſcribed, or whether from the whole ſide one Perimeter be deſcribed. Now the Superficie of a Cylinder (that is, the thin Coat of the Cylinder ſtript of, its Skin, being diſplayed and extended in a Plane) becomes a Triangle, of which one ſide is the Diameter of the Baſe, the other a ſtreight Line, whoſe Square is equal to ten Squares from the Diameter of its own Baſis.

CHAP. X.

Of a new Method of Treating of Solids and their Superficies, by the efficient Cauſes.

LEMMA.

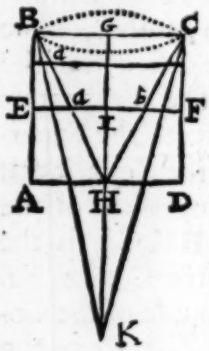
AN Agent working Uniformly in a given time, for perfecting a deſigned work, if in ſome parts of that time he work, and in ſome others reſt; the work that is done, will be to the work left undone, as the time wherein he worketh, to the time wherein he reſteth. For example, If a given piece of Land may by continued labour be wholly Plowed in three days; then if it be only Plowed two days, and one day ſpent idlely; that which ſhall be Plowed, will be to that which remains unplowed, as 2 to 1; and what was deſigned to be Plowed, will be to what is Plowed, as 3 to 2, and to the unplowed, as 3 to 1. In like manner, how much an Agent by continued and uniform labour can do in any time, a double Agent by the like labour can do

can do as much, if one half of the time be ſpent in work, and the other half idlely.

Moreover how much the Agent loſeth in time of working, ſo much it muſt be thought he reſted, and theſe are manifeſt by the light of nature.

1. The Convex Superficie of a ſtreight Cylinder, is equal to the Superficie of a ſtreight Cone, having the ſame Baſe with the Cylinder, but double the height.

Here (Fig. 1.) take the Cylinder ABCD, ſuch as is propoſed, whoſe Baſe is the Circle BC. Now ABCD is a Square, let the Square ABCD be divided in two in the middle by the right Line GH, parallel to the ſide AB, and lengthen GH to K; let then GK be the double of GH. Joyn BK, KC, and BH, CH; then the Square ABCD, and the Triangle BKC have the ſame Baſe, but the height of the Triangle BKC, is double the height of the Square ABCD. Suppoſe the right Angle DG to turn round about the Axis GH, and by that Circumvolution from the Rectangle DG a right Cylinder will be deſcribed, but from the Triangles CHG, CKG, two right Cones, and from the ſide DC, the Superficie of the Cylinder, and from CH, CK, the ſides of the Cones, two Conical Superficies, and from the Point C, the circumference of the Baſe. Theſe things ſuppoſed we are to demonſtrate that the Conical Superficie of the Cone BKC, is equal to the whole Superficie of the Cylinder ABCD.

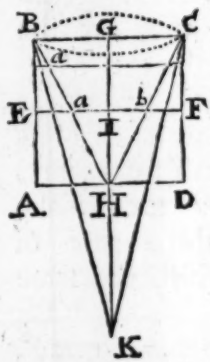


Draw the right Line *ab*, as you Pleaſe, but parallel to the ſide BC, cutting the ſtreight Lines BH, CH

in *a* and *b*. The Proportion then of BC to *ab*, will be the ſame with the Proportion of H*a* to HB; and becauſe *ab* is placed any where, the ſame will the Proportion be every where; and becauſe the right Line BC by an uniform motion, and to right Angles by the ſides BA, CD, will compleat both the Cylinder ABCD, and its whole Superficie; but the motion failing according to the Proportions of times, they will deſcribe the Triangle BHC; the right Line BC reſts as much from working, as it worketh. The Superficie BHC will then be equal to the two Superficies BAH, CDH, that are not made (according to the foregoing Lemma.) Now becauſe the Diameters are in the ſame Proportion one to another, as the Perimeters; the Perimeters alſo will be deficient in the ſame Proportion with the times. The Perimeters therefore of the Circles, which conſtitute the Superficie BHK, that is, the Superficie of the Cone it ſelf BHC, is equal to the half of the Superficie of the whole Cylinder. But the Triangle BKC, is the double of the Triangle BHC, and the Conical Superficie of the Cone BKC, is the double of the Conical Sperficie of the Cone BHC. Therefore the Superficie of the Cone BKC, is equal to the Convex Superficie of the Cylinder ABCD, which is the thing propoſed.

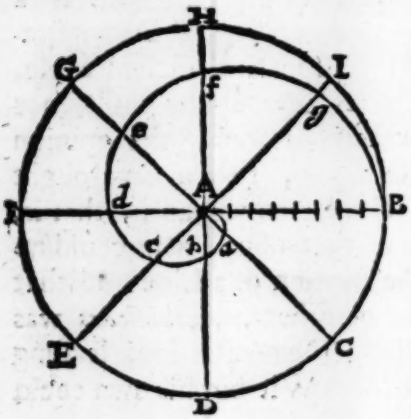
2. The Convex Superficie of a Cylinder, is the Quadruple of the Baſe of the ſame Cylinder; for ſeeing the Convex Superficie of a Cylinder, is made by the Perimeter of the Circle, which is the Baſe of the Cylinder working uniformly; if a Perimeter which is the double of the Perimeter of the Baſe, ſhould work one half of the ſame time, and reſt another half, it will perform as much (according to the foregoing Lemma.)

Lemma.) But the Perimeter of the Circle, whoſe Radius is H G is the double of the Perimeter of the Baſe B C; and the Circle from H G, is made by the Radius H G equal to B C by a circular Motion, that is, the Radius B C ceaſing as much as it operateth, to wit, failing in operating according to the Proportion of the times wherein it doth operate. Therefore the Circle, whoſe Radius is H G (which is the Quadruple of the Circle that is the Baſe of the Cylinder) is equal to the Convex Superficie of the Cylinder A B C D. And this is a ſhort, clear and natural Demonſtration drawn from the efficient cauſe, but ſuch as *Archimedes* could not make uſe of; it being the opinion in thoſe times, that a Line ought always to be conſidered without any breadth, that is, as nothing; and therefore that a Superficie could no ways be deſcribed by the Motion of a Line; ſo that becauſe of the prejudices of others, *Archimedes* was forced to make uſe of a long Demonſtration, leading *ad impoſſibile*, which no Mans Wit but his own could have overcome.



3. A right Cylinder is the triple of a right Cone inſcribed in it. For a Cylinder is made by the uniform Motion of the Circle B C (I ſay of the Circle, not of the Perimeter) which is the common Baſe both of the Cylinder A B C D, and the Cone B H C. Now Circles have a double Proportion of that which their Diameters have. Therefore the Circle B C, to the

Circle *a b*, is in a double Proportion of the Diameter B C to the Diameter *a b*, and ſo every where. Now the Circle whilſt it maketh the Cone B H C, loſeth every where of its Magnitude in working, the double Proportion of the Diameter B C to the Diameter *a b*. But (as has been ſhewed chap. 2. art. 9.) when mean Proportionals, as well Geometrical as Arithmetical are taken every where, all together they are the ſame. Therefore the Circle B C loſeth of its Magnitude, whilſt it maketh the Cone B H C, two thirds of its own entire Magnitude. Now how much of its Magnitude it loſeth, ſo much it ceaſeth from work. It will then make the third part of that which the ſame entire Circle B C, would have made, that is, a third part of the whole Cylinder. Therefore a Cone is the third part of a Cylinder, that is, a Cylinder is the triple of a Cone inſcribed in it.



4. It may be ſhewed by the ſame method, that the ſpiral ſpace which ſprings from the firſt revolution of a Circle, is the third part of the ſame Circle.

From the Centre A, by the Radius A B, let a Circle be deſcribed, and eight ways cut by the ſtreight Lines A C, A D, A E, *&c.* Divide likewiſe the Semidiameter eight ways, of which let one eighth A *a* be marked in the ſtreight Line A C, then two eighths in the Radius A D at *b*, three in the Radius A E

at *c*, four in the Radius AF at *d*, five in the Radius AG at *e*, ſix in the Radius AH at *f*, ſeven in the Radius AH at *g*. Now you muſt ſuppoſe that every eighth part is divided, as the whole Radius, in as many equal parts, as it can be ſuppoſed it can be divided into; ſo that of what parts AB is 8, A*a* is 1, A*b* 2, A*c* 3, A*d* 4, A*e* 5, A*f* 6, A*g* 7; then you muſt ſuppoſe a ſpiral Line (whoſe beginning is A, and end B) drawn through all the Points, *a*, *b*, *c*, *d*, *e*, *f*, *g*, B; and this will be the ſpiral Line deſcribed by *Archimedes*. We muſt then ſhew, that the ſpace concluded by this ſpiral Line, and the Semidiameter AB, is the third part of the Circle BCD—

Becauſe the Circle BCD, is made by the circumduction of the entire Radius AB, and the ſpiral ſpace by the ſame Radius, but failing, that is, reſting every where according to the double Proportion of the times (for all the Circles through *a*, *b*, *c*, *d*, &c. have a double Proportion of the ſtreight Lines A*a*, A*b*, A*c*, &c.) the ſpace which is left without the ſpiral Line unmade, will be the double of that which is made by the ſpiral converſion; and therefore both the ſpaces, made and unmade together, are the triple of the ſpiral ſpace.

5. Now it is manifeſt, that ſince the ſpiral Line A, *a*, *b*, *c*, *d*, *e*, *f*, *g*, B, is contually deficient in the ſame Proportion with the times, it is equal to the half Perimeter of the Circle BCD.

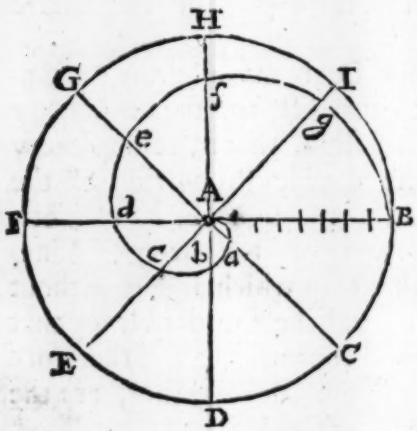
6. That a Solid alſo whoſe Baſe is the ſpiral ſpace, is the third part of a Cylinder whoſe Baſe is the Circle BCD—

7. Alſo that a Square circumſcribed by the Circle BCD—is to the ſpiral ſpace as 15 to 4. Now (as

We

we Demonſtrated before) a Square is to a Circle inſcribed in it, as alſo a Cube from the Radius, to the Cylinder, as 5 to 4. The Proportion then of a Square to the ſpiral ſpace, is compounded of the Proportions of 5 to 4, and 3 to 1; ſo that if it be, that as 3 to 1, ſo 4 to another, which ſhall be $\frac{4}{3}$: The Proportion of a Square to the ſpiral ſpace will be made up of the Proportions 5 to 4, and 4 to $\frac{4}{3}$; the Square then is to the Spiral ſpace, as 5 to $\frac{4}{3}$, that is (both being multiplyed by 3) as 15 to 4.

8. It likewiſe follows from the ſame, that if the Circle BCD — be the Baſe of a Cylinder, whoſe Baſe is equal to the Diameter of the ſame Circle, and that there be a Solid, whoſe Baſis is indeed the ſpiral ſpace, but the height equal to the height of the Cylinder; the Sphere wherein the greateſt Circle is BCD, will be the double of the ſaid Solid. Now we have ſhewed before, that a Square into which the Circle BCD is inſcribed, as alſo a Cube from the Diameter, is to a Cylinder whoſe Baſis is the Circle BCD, as 5 to 4, and to a Sphere, as 15 to 8, and to the ſpiral ſpace, as 15 to 4; for the Proportion of 15 to 4 being ſubtracted from the Proportion of 15 to 8, there remains $\frac{60}{120}$, that is, $\frac{1}{2}$. The Solid then whoſe Baſis is the ſpiral ſpace, is the half of the Sphere wherein BCD is the greateſt Circle.

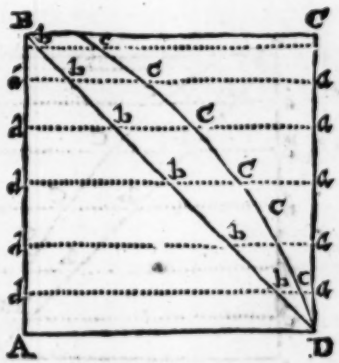


9. By

9. By the ſame method it will be found, that any Rectangle is to the Parabole inſcribed in it, as 3 to 2.

Deſcribe then any right Angle A B C D, in whoſe ſide D C take any where D *a*, to which in the ſide A B, take an equal A *d*, cutting the Diagonal B D in *b*, and let that be ſo every where; then as *d a* every where to *d c*, ſo let *d c* be to *d b*, and draw the crooked Line B *c c* D through all the Points *c*; that this crooked Line is Parabolical, and the Figure A B *c c* D the half of a Parabole, all Mathematicians do agree. It is then to be ſhewed, that the Rectangle A B C D is to the half Parabole A B *c c* D, as 3 to 2.

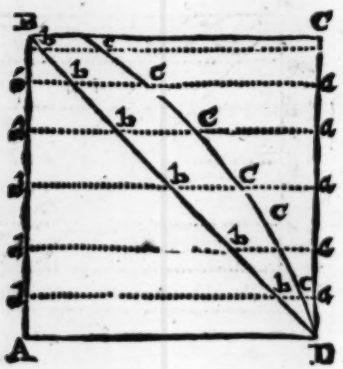
For deſcribe the Rectangle A B C D from the right Line A B uniformly, and parallelways moved to the oppoſite ſide D C. Now the Semiparabole A B *c c* D is deſcribed by the right Line C B moved towards the oppoſite ſide A B, ſtill loſing in the double Proportion of the time A B to the time D *a*; for the Proportion of *d a* to *d c* is every where the double of the Proportion of *d a* to *d b*; and therefore (according to chap.2. art. 9.) all the *d a* together, are to all the *d c*, in a double Arithmetical Proportion of all the *d a* to all the *d b*. So then that which is made uniformly from the whole AB(that is,the whole rectangle ABCD) to that which is made by the Motion of the ſide DC towards AB failing or loſing (that is, reſting from work)



work) according to the ſubduplicated Arithmetical Proportion of time, will be that which is left unmade, to wit, CD*cc*B one part of the whole Rectangle ABCD, of which that which is perfected, to wit, the half Parabole A B*cc*D is two.

And ſo the Rectangle ABCD, is 3 of the ſame parts: The Rectangle ABCD is therefore to the Semiparabole inſcribed in it, as 3 to 2, and the double Rectangle to the whole Parabole inſcribed in it, as 3 to 2.

10. We may alſo uſe the ſame method in finding out the Proportion of a right Angle to the Parabolaſter (which is called a Cubical Parabole) for if it be every where that as *da* to *dc*, and *dc* to *db*, ſo *db* to a fourth *de* (ſo that there be four continual Proportionals *da*, *dc*, *db*, *de*) the crooked Line of that Parabolaſter will paſs through all the Points *e*; and the Proportion of the Rectangle ABCD will be to the half of that Parabolaſter, as 4 to 3; and ſo you may proceed to the ſecond, third, *&c.* Parabolaſter, whoſe Proportions to the Rectangle in which they are inſcribed, (as alſo of Cylinders to the Cone and Conoides inſcribed in them) are reduced into the table of chap. 17. of my Book *de corpore*. Such is then the natural aptitude of the *Lemma* prefixed to this Chapter, to the Demonſtrating of the greateſt Problems of pure Geometry,



Geometry, that he who knows by what Proportion of Proportions (for ſeeing Proportion is a Quantity, there will be Proportions of Proportions, as well as of other Quantities) every compared Figure is framed, he cannot be ignorant of their Proportions one to another. But what (may ſome body ſay) is that pure Geometry? is that Geometry true which is impure? Let him term it mixt, not impure. But how can a thing pure, mixed to a thing pure, become impure? but neither muſt that be ſaid; but that then it is mixt, when it is applyed to matter. For it is indeed manifeſt, that if it be not applyed to matter, it is uſeleſs, and no more but meer hard words. I call that pure Geometry, with which nothing of Arithmetick is blended, but what may alſo agree to a continual Quantity, which is the proper and adequate Subject of Geometry. Therefore the chief things that corrupt Geometry, *are the ſurdity of Numbers, Longitude without Latitude, and Latitude without thickneſs*, and the lately introduced Doctrine, that Fraction is Proportion, as if the half were the Proportion of the half to the whole.

CHAP.

Chap. XI.

Of Demonſtration.

IN the firſt place it may be doubted, whether Demonſtration goes before, or comes after Knowledge. Moſt ſay, that it goes before; for a Demonſtration is either Scientifick or frivolous: it is therefore the efficient cauſe of Knowledge, and for that reaſon goes before its effect. It muſt then be granted, that the Maſters Demonſtration, goes before the knowledge of the Scholar; but no more. But if any concluſion be Demonſtrated by any Man, he muſt firſt have known the truth of it, before he could Demonſtrate it either to himſelf or others. For no man can Demonſtrate that which he knows not whether it be falſe or true: It is then manifeſt, that knowledge in its own nature goes before Demonſtration.

In the next place it may be asked (ſince Geometry is very uſeful and ornamental to mankind) who are chiefly the men to whom we are indebted for ſo great a good, the Demonſtrators or not Demonſtrators. It is certain that long before *Euclid*, many large and artful Fabricks were built, the Tower of *Babylon*, the Pyramids of *Egypt*, the wonderful Walls and Gardens in *Babylon*, the Palace of *Perſia*, and others; alſo a Sundial was firſt ſhewed at *Lacedemon*. Theſe works without doubt required knowledge; yet the Authors of them Demonſtrated nothing, but by natural Logick foreſaw the reality of their future works, though others afterward were curious to do it, with that purpoſe

purpoſe, that they might excite the minds of many, to the invention of uſeful things. The Proportions alſo of the five regular Bodies (as you may read in the old *Greek Epigram*) were found out indeed by *Pythagoras*, though *Plato* Demonſtrated them, and after him *Euclid El.* 3. *Plato*'s Demonſtration is not extant; ſo that we owe thoſe good inventions to ſeveral Inventors, whoſe names (except a few) are now loſt, and not be Demonſtrators. How then? is Demonſtration uſeleſs? not indeed to thoſe who are taught: Now to be taught I think not very laudable, though to teach, provided it be rightly done, and without hire is honorable. But doth not a Demonſtration of ſome ample Science hitherto unknown, deſerve great thanks? Yes, but he that ſhall do that, is to be reckoned among the Inventors of profitable things. And therefore we are indebted to *Euclid*, who firſt of all taught the World the method of Demonſtrating, that is of ſound reaſoning. All agree that moſt ample thanks are due, to thoſe who firſt adviſed men to aſſociate, and to unite together under the obedience of one Supream Power. The next I think is due to them, who ſhall perſwade Men that they ſhould not violate the Compacts and Agreements that once they have made.

Thirdly, Some may ask what a Demonſtration is; moſt men, and not without ground uſe to call a Demonſtration, an evident Probation of the truth, in any dubious Queſtion. Every one thinks that he has enough of Demonſtration, when his mind does fully acquieſce to thoſe things that are alledged for Probation. But the ancient Philoſophers nurſed up in perpetual Diſputations, as often as there was any debate concerning the Compariſon of Motions and Magnitudes, for

for moſt part deſcribed Figures, and put before the Eyes of their Diſciples, as if ſhewed by them to the ſight (that is, ſomewhat more as they thought than proved) called their arguments ἀποδείξεις, (in Latine, Engliſh) Demonſtrations. But now when any man is a little too vehement and poſitive in aſſerting, he affirms that he has Demonſtrated.

Now a Demonſtration is when the truth of the concluſion hath its firſt foundation, in thoſe things which are already known to them, to whom he that proves them ſpeaks. Now theſe Foundations, are Definitions, and beſides Axioms which are not indeed Demonſtrated, yet ought to be true and well underſtood; for from truths nothing but what is true can be inferred, nor can any thing though never ſo true be known, that is not underſtood.

Fourthly, It may be demanded what the uſe of Definitions and Axioms is: This is the plain uſe of Definitions, that he to whom a Demonſtration is made, may underſtand what certain and univerſal ſignification of the word, the Demonſtrant would have taken; for the ſignification that changeth deceiveth. Therefore a Definition conduceth to the underſtanding, and without underſtanding there is neither any Demonſtration, nor in him that learneth can there be any knowledge, but from thence unſeaſonable and obſcure diſtinctions are introduced, ſo that of what is ſaid, nothing maketh impreſſion on the imagination. There are indeed ſome truths which can no ways be known by man, but nothing is Demonſtrated that cannot be underſtood by him.

The uſe of Axioms conſiſts in this, that they abbreviate the too long Series of Demonſtrations, to wit, by removing unneceſſary Demonſtrations. For the truth of

of Axioms ought to appear ſooner and more clearly than the means themſelves, by which they are proved.

In every Demonſtration, the cauſe of the concluſion ought to be in the Antecedents, by virtue of which it is inferred, that is, in things before Demonſtrated, or known by the light of nature. Therefore where the words cohere and hang together, there will be a Demonſtration; for though the cauſe of the ſubject matter be not known, yet the concluſion will retain the truth of the Principles from whence it is derived. Now Demonſtrations of this kind are eaſie, though they little advance knowledge.

That is the chiefeſt of all Demonſtrations, which is drawn from the production of the Subject matter, according to the order of Nature; and ſo theſe are the moſt uſeful Definitions for knowledge, wherein the generation of the ſubject matter is explained, that is, by what motion, what concourſe of motions, what proportions of motions and times all ſpaces and magnitudes are determined.

The next Demonſtration to this, is when (from the negation of truth) ſomething impoſſible is inferred. Now this kind of Demonſtration has its force from this, that from a truth nothing but truth can be deduced.

Chap. XII.

Of Fallacies.

That a Man who underſtands his Words, that is, his own Speech, and ſets upon the proof of it from true Principles (I mean) natural Definitions, proceeding ſlowly in Mathematical matters, ſhould be long and often deceived, and being admoniſhed ſhould perſiſt in his errour, is a thing almoſt impoſſible; ſpeaking properly, there can be no errour in the intellect; for to erre in the intellect, is the ſame as not to underſtand. The right uſe of the Tongue, Feet and Hands is not Demonſtrated by a Maſter, but by exerciſe we learn it; and though all our words almoſt change their ſignification, according to the variety of the things whereof we ſpeak or write; yet we uſe them at home, in the Fields, and in the Market without any hurt, becauſe it is enough for civil Society, if one underſtand aright what another ſays.

It is otherwiſe in Philoſophy, where nothing but truth is ſought after; but eſpecially if glory attend invention. It is one thing to walk, another thing to walk upon a Rope; the one is eaſie and without great damage, though a man ſhould trip; ſo that negligence is there pardonable, it is not ſo to a Rope dancer. Truth walks upon a very ſmall thred, without that Metaphorical Latitude of common converſation; and eſpecially Mathematical Truth, which unleſs it be poiſed by the weight of Definitions and Axioms, it tumbles headlong to the laughter of Spectators.

The

The chief and moſt frequent cauſe then of Fallacies in the Mathematicks is, that they build their reaſoning upon Definitions not underſtood, or falſe or ambiguous ones, from which no truth can be deduced. The Greek Philoſophers were of the ſame mind alſo, when to this Science they gave the name *Mathematicks*, from the Greek word that ſignifies to underſtand; for commonly when any one ſpeaking to another, doubted if he was underſtood or not, he asked, μανθάνεις, *do you underſtand*, to which it was anſwered, μανθάνω, or ὐ μανθάνω, that is, *I do*, *or do not underſtand*; ſo natural it was to denominate the Mathematicks from the underſtanding. Another cauſe of Fallacies is, not to know what motion is, and its properties, that is, to be ignorant of the immediate natural cauſe of all things. Thoſe known Fallacies reckoned up by *Ariſtotle*, by which a Child can hardly be deceived, I purpoſely paſs over. As the greateſt bane of Geometry, in the firſt place, I condemn a *Line without Latitude*, a thing unconceivable. Secondly, the ſide of a Square Figure ſuppoſed for the root of a Number. Thirdly, the nature of Proportion not underſtood. Fourthly, all conſideration of *infinitum*, whether Geometrical or Arithmetical. And here I ſhould have made an end, had not there been one who affirms that he can alſo Demonſtrate thoſe things which are neither intelligible, explicable, nor conceivable; which is indeed to ſay, that all Sciences are not worth a ruſh.

Nothing (ſays he) *is more obvious in nature than continual Quantity, and local Motion. Now theſe either are, or are not diviſible in* infinitum; *can this disjunctive be denyed? can it be ſaid, that they neither are, nor are not thus diviſible? let any chooſe what member of this he pleaſes; ſhall he remove the difficulties that are in it? or*

ſhall be anſwer the Objections that can be made to the contrary.

Though theſe be Queſtions, yet in this place they have the force of negations; let him then object his difficulties. I ſhall explain the matter it ſelf, (to wit) that which Mathematical Writers underſtand when they ſay, that Quantity is diviſible *in infinitum*. They do not ſay nor underſtand, that a finite Quantity (ſuppoſe a Line) is diviſible into parts, that are infinite in number; but that a Line never ſo little is of its own nature capable of diviſion. Neither by diviſion do they underſtand a material Separation, that is, a Separation of one part from another; but that in all continual Quantity, a Quantity leſs, and that aſſignable is ſtill to be ſuppoſed; and that the ſignification of this Diviſion is nothing elſe but the unlimited conſideration of a part in a whole never ſo little. For of whatſoever it may be ſaid, it is a whole, it may very well be ſaid that there are parts in it; but that a mortal man can divide any thing eternally, or if he could do it, that yet the parts ſhould not be of a finite number, it is impoſſible. Now what is ſaid of a Line, ought likewiſe to be underſtood of Motion, Time, and every thing that is diviſible, except Number. Let an Objection now be brought againſt this, (I confeſs I do not remember that ever I read any) that we may ſee if it may not eaſily be underſtood whether it be ſtrong or weak. Moreover,

I ſuppoſe (ſays he) *that moſt know, that famous argument of* Zeno, *which is called* Achilles; *and that how that great diſputer, whil'ſt that by apparent impoſſibilities, and abſurdities on each ſide, he Demonſtrated local motion to be impoſſible, he was confuted by one of the heavers, who roſe and walked through the School.*

Of

Of which words this is the Summ, That *Zeno* was indeed confuted by the Walker, but that his Sentence was rightly inferred by a Demonſtration leading to an abſurdity. What *Zeno*'s Argument is, that the Reader may judge of it, I will ſhew. Seeing the leaſt ſpace that is, cannot be paſſed over by motion, but that the half of that ſpace is firſt to be paſſed over, and that the leaſt ſpace again has alſo its half, and ſo perpetually; *Zeno* concludes (ſubſuming that no ſpace can be paſſed over in an inſtant) that it requires an infinite time to paſs over the leaſt ſpace whatſoever. Now it is manifeſt, that if a ſpace cannot be paſſed over without an eternal motion, it cannot at all be paſſed over.

The Stoicks exemplified this Argument taken from *Zeno* their Maſter in *Achilles* ſwift of Foot, and a ſlow-paced Tortoiſe; and they ſaid (putting *Achilles* and the Tortoiſe into the race) that *Achilles* could never overtake it, if it were but the leaſt diſtance before him. But why? becauſe whil'ſt *Achilles* ran over that half diſtance, the Tortoiſe alſo advanced a little. I ſhould be aſhamed to repeat thoſe trifles, did I not perceive, that they who leaſt ought to be, may yet be deceived by ſuch Childiſh Fallacies: But what is to be anſwered, I ſhall anſwer, Firſt, That it is no wonder if he that is unwilling, never overtake a ſlow runner that is got before him. For ſuch is the nature of motion, that he who always will, or is forced to ſlacken his ſwiftneſs in Proportion to the ſpace that is left, will never be able to paſs over even the ſmalleſt ſpace whatſoever. Secondly, what is made by any whole, and the half of it, and the half of that half, and ſo on, will be leſs than two; they are ſo far from making an infinite time. By that Argument of *Zeno*'s then,

it cannot be inferred that ſwift *Achilles* could not in running overtake the Tortoiſe; but only this, that if he pleaſed he would not. Therefore the example of *Zeno*'s Sophiſme does not prove that there can be a Demonſtration of a thing that is not intelligible. He goes on.

The pure and moſt ſimple Mathematicks are the moſt eaſie, evident and comprehenſible of all humane Sciences. Yet in Geometry and Arithmetick, how many Propoſitions are there firmly Demonſtrated, which nevertheleſs are inexplicable, unconceivable and incomprehenſible? I ſhall give a few inſtances, &c.

Is not this abſurd? can he be ſaid to Demonſtrate a truth that does not make it apparent and intelligible? how can I know whether it be true or falſe, if I cannot imagine it in my mind? but let us read the examples which he ſays he will ſubjoin.

1. *That the leaſt ſpace imaginable may be equal to another ſpace upon the ſame Baſe, and of the ſame height, whoſe ſides are drawn out* in infinitum.

That a Parallelogram and Triangles, indeed, of the ſame height, upon the ſame Baſe, are all equal to one another, is intelligible to every one. What he adds, *whoſe ſides* are drawn out *in infinitum*, is abſurd; as a bench is not called a bench before it be perfected, ſo neither is a ſpace before it be finiſhed.

For it is down right impoſſible and abſurd; nor from *Torricellius* does it follow, that a finite is equal to an infinite.

2. *One infinite may be greater than another infinite:* Who ſaid ſo? *he is ſilent, for it is abſurd.*

3. *That all the circular Angles of contact are equal; what is a circular Angle of contact? the expreſſion it ſelf is unintelligible.*

4. *The*

4. *The eternal appropinquation of two Lines, but the concourſe impoſſible, to wit, Aſymptotes. The cauſe of this is before explained in the Sophiſme of* Zeno.

5. *The affections of ſurd and irrational Quantities; all theſe are ſo Demonſtrated, as that they cannot be denyed, and many others, which nevertheleſs are inexplicable, incomprehenſible and inconceivable.* But what are theſe ſurd and irrational Quantities? that there are ſome continual Quantities, which have not the Proportion of number to number, and are called incommenſurable, there is no Mathematician but knows; and of theſe alſo that are commenſurable one with another, many are irrational; becauſe though they be commenſurable one with another; yet becauſe they are not commenſurable to any Quantity taken at pleaſure, they are ſaid to be irrational.

6. *In Numbers can the affections of a unity and ternary be fully comprehended, expreſſed and explained, that in nature there ſhould be one Quantity, and no other, which with its infinite powers aſcending, and roots deſcending, all ſhould be equal among themſelves, or rather one and the ſame?* The words themſelves are pretty obſcure; but I think this is it, that he would have ſaid. Seeing the ſquare power of a number, and a ſquare number; alſo the ſecond power and cubical number, *&c.* is the ſame thing to Arithmeticians; and that that number is called the Root of a Square Number, which being multiplied by it ſelf makes any number, and the Root of a Cubick Number the ſame with that Number, which being multiplyed by it ſelf, and again by the product maketh any one, and that one multiplyed by it ſelf never ſo often, makes no more but one; it is manifeſt that all the products and their Roots are the ſame Unities. Now though this ſecret be only proper to an unity,

yet it is not hard to be underſtood, becauſe nothing can be multiplyed by one. Laſtly,

He adds, *Can ſo much as any one of the infinite potential Roots of the Number* 3, *be explained, thought, or comprehended?*

It cannot, I know; and why? neither that, nor that of many other numbers, can, I know, and can briefly explain, and that is, becauſe they have no Root.

Chap. XIII.

Of an Infinite.

TO the Word *infinitum* ſomething is underſtood as work, time, ſpace, *&c.* if it ſignify a work, then in its Latine acceptation it ſignifies a work that is indeed begun, but not as yet brought to the end that the Artiſt deſigned in his mind; and ſo in that ſenſe, a thing infinite is the ſame as a thing unfiniſhed or imperfect, but what may be finiſhed? Such as a houſe begun, but not perfected. If any then ſhould ſay of an Artiſt, that he had in his mind to perfect an infinite work, he would ſpeak abſurdly, as if he ſhould ſay, that it was in his mind to do that which he never intended: For no man thinks of doing more than what he can accompliſh; ſo that no man can judge of a work that is not his own, without conſulting the Artiſt, whether it be finiſhed and perfect, or infinite, that is, unfiniſhed and imperfect.

An

An *Infinite*; if ſpace be underſtood, ſignifies a ſpace greater than can be equalled by the greateſt number of meaſures, as of Feet, Paces, Miles, or even of the Diameters of the Earth, or of the Orb of the fixed Stars, that is to ſay, which cannot be included within bounds. In like manner an infinite time is that which no number of hours, or days can equal.

Therefore of an *Infinite*, according to this ſenſe it cannot be ſaid, that one is greater than another: Draw then the finite right Line A B, and ſuppoſe it produced beyond D A C B E B by E *in infinitum*. Therefore both the right Line B E —— and A B E —— are infinite in length. Now A B E —— is greater than B E —— by the whole length of A B, be it ſo. Let A B be divided in C, and put A D equal to A C; and let it be ſuppoſed lengthened ſtrait ways by F *in infinitum*. A D F will be then greater than D F by the finite Longitude C D, that is, by the Quantity of A B.

Wherefore C B E —— and C A D —— are not unequal; therefore there is ſome certain Point, there is in the infinite Line —— D B —— a mean, ſo that the Centre of an infinite Sphere will be the Point C, and (becauſe the Points A and B are taken at pleaſure) in every point of an infinite Sphere, will the Centre of the Sphere be, and ſo the Semidiameters of an infinite Sphere from any Centre, whether A, B, or C, are not unequal one to another. Therefore one infinite Line is not greater than another.

By the ſame reaſon, if the Line A B be put for an

an infinite time, it may be proved that two eternals cannot be unequal.

Now the cauſe why a finite may be conſidered in an infinite, conſiſts in this, that neither the ſubject Body is in the thinker, nor ſpace, the Image of the Body in the thing thought of, but only in the memory. Now in the memory and ſenſe there can be no infinite.

But Mathematicians uſe often the Word *infinite* for *indefinite*. Now indefinite is the ſame as *never ſo great*, and ſometimes it is taken for *infinitely little*, provided it be not *nothing*. Sometimes alſo *infinite* goes for *as much as is poſſible*. But nothing is properly *infinite*, unleſs it exceed all aſſignable number of given meaſures. But it is ſaid to have been Demonſtrated by *Torricellius*, that a certain acute Hyperbolical Solid, is even in this ſenſe of *infinite*, equal to a certain Cylinder, whoſe Baſe hath a Diameter equal to the half Baſe of the Hyperbole, but a height equal to the tranſverſe Axis of the ſame Hyperbole. I have often and attentively read the Demonſtration of this Problem, and never found any Sophiſme in it. Yet I found that the diſtance which *Torricellius* ſuppoſes infinite, is meant of an indefinite diſtance; nor could it be otherwiſe underſtood by himſelf, who in very many Demonſtrations uſeth the Cavallerian Principle of indiviſibles; which indiviſibles of *Cavallerius* are ſuch, that their aggregate may be equal to any given Magnitude. So that ſo abſurd a Propoſition as this, *an infinite is equal to a finite*, ought not to be aſcribed to *Torricellius*, for there can be no ſolid ſo ſmall, which doth not infinitely exceed every finite ſolid, as is manifeſt by the

light

light of Nature; that is, the abſurdity of Arithmeticians reaſoning about an infinite, and meaſuring Superficies and Solids by Lines without Latitude, who obſerving no difference betwixt Arithmetick and Geometry; took the Root of a Number (which is part of its ſquare Number) for the ſame thing with the ſide of a ſquare Figure, though they confeſs that the ſide is no part of its own Square: So much of Mathematicks. I now expect what the Algebraiſts will ſay to the contrary.

The End of the Third Book.

MEASURING

OF

Glazing, Painting, Plaistering, Masonsy, } { Joynery, Carpenters, and Bricklayers,

WORKS.

BY

Vulgar ARITHMETICK

without Reducing the Integers into the leaſt Denomination.

The Fourth Book.

By *VEN. MANDEY.*

LONDON,
Printed Anno MDCLXXXI.

MEASURING OF CARPENTERS WORK.

CHAP. I.

Menſuration,

OR Meaſuring is a Science, whereby we are certified (in ſuperficial Meaſure) either how many Inches, or how many Feet and Inches, or how many Yards, Feet and Inches. Or how many Squares, Feet and Inches there is in any Dimenſion, whether it be of Glaſs, Board, Stone-paving, Plaiſtering, Painting, Tyling, Carpenters or Joyners Work, *&c.* And ſo conſequently it certifies us of the Content of many Dimenſions, by adding the Products of the ſeveral Dimenſions together.

It doth the ſame in ſolid Meaſure; there is only this difference between Superficial and ſolid menſuration (to wit) in Superficial Meaſure there is only two Summs in a Dimenſion (*viz.*) Length and Breadth to be multiplied one by the other. But in ſolid Meaſure

ſure there are three Summs in a Dimenſion (to wit) Length, Breadth, and Thickneſs to be multiplied each by other, and this kind of ſolid Menſuration ſerveth for the Meaſuring of Timber, Stone, Digging, Bricklayers Work, (for it is commonly reduced to a thickneſs) and all manner of ſolid Bodies whatſoever.

The Inſtruments that are uſed in taking of the Lengths and Heighths, (or Breadths) in Meaſuring of the following Works; are a Ten Foot Rod, and a Five Foot Rod, and a Two Foot Ruler, and ſometimes a Line.

I ſhall begin with the Meaſuring of Carpenters Work, it being moſt eaſie to learn, and ſo proceed through the other Trades, leaving the Meaſuring of Bricklayers work until the laſt, it being moſt difficult.

Chap. II.

The Flooring, Roofing, Partitions, and Ceiling Joiſts, being Carpenters Work, is generally Meaſured by the Square (as it is vulgarly ſaid) or more properly is reduced into Squares.

Which Square conſiſts of (or contains) 100 ſuperficial Feet, being the Product of a ſquare Superficies multiplied in it ſelf, being 10 Feet in Length, and 10 Feet in Breadth.

Note, That a Superficies is that which hath only Length and Breadth, as a Square encloſed with four Lines on Paper, is called a Superficies (as appears more at large in the precedent Treatiſes of Geometry.)

And

And from thence the Measuring of Flooring, Roofing, Partitioning, Ceiling, Joisting, Boarding, Plaistering, Painting, *&c.* is called superficial Measure, because their thicknesses are not considered, nor taken notice of in measurement.

For generally Boards, Glass, Painting, Plaistering, *&c.* are severally for the most part of one thickness.

And although some Floors, and Roofs, and Partitions, do require to be made stronger than othersome, by reason of the greatness of the Building, and by this means the Timbers are larger and thicker, yet there is no respect had to, (nor cognisance taken of) the largeness and thickness of the Timbers, in the measurement: But an allowance is made in the price by adding so much per Square more than is usually given for ordinary work.

Thus 12 Inches in Length, and 12 in Breadth is called a Superficial Foot, of Roofing, Flooring, Boarding, Partitioning, Glazing, Painting, Plaistering, *&c.*

Note, When a Carpenters Bill of Measurement is made, there is set down,

For so many Squares of Roofing (at what price they agree upon per Square) so much money.

Likewise for so many Squares of Flooring and Boarding, at so much per Square, so much money.

Also for so many Squares of Partitioning at so much per Square, so much money.

And for so many Squares of Ceiling Joists, *&c.*

The Windows they set down either at so much per Light, or so much per Window.

The Doorcases at so much a piece, either with, or without Doors.

The Mantletrees and Tarsells at so much a piece.

The

The Lintolling, Guttering, Cornice, and Window Boards, at ſo much per Foot.

Stairs at ſo much a Pair, or ſo much per Step, &c.

CHAP. III.

IT is ſuppoſed that he that intends to learn the Science of Meaſuring is an Arithmetician.

For although ſeveral Propoſitions may be anſwered, ſome Geometrically, and ſome on the Line of Numbers, (otherwiſe called *Gunter*'s Line) and ſome on the Lines of Superficies and Solids on a Sector; yet without Arithmetick it is impoſſible to Meaſure exactly all kinds of Plains and Bodies.

Therefore to the Arithmetician, I ſay, multiply the Length of any Dimenſion (that is, either Square or Oblong) by the Breadth thereof, and the Product is the Superficial Area or Content.

I ſhall begin with a Dimenſion of Inches, and ſo proceed gradually to Feet, and from Feet, to Feet and Inches (or parts).

Note, That two ſides of a Superficies being meaſured and expreſt by Arithmetical Figures (one ſumm being the length, the other the Breadth) is called a Dimenſion.

Example in Inches.

Suppoſe a piece of Board or Glaſs, or flat Stone, or Painting, or Plaiſtering, or any thing, be it what it will, that is to be meaſured Superficially, (vulgar-

ly called flat Measure) be 11 Inches in Length, and 9 Inches in Breadth (which two summs 11 and 9, are called a Dimension) and you would know the Area or Content thereof.

	In.
Set down your Dimension thus —	11 the length.
	09 the breadth.
Then multiply the Length by the Breadth.	
And the Product is———	99 Inches.

Which 99 Inches is half a Foot and 27 Inches, or very little more than $\frac{2}{3}$ of a Foot.

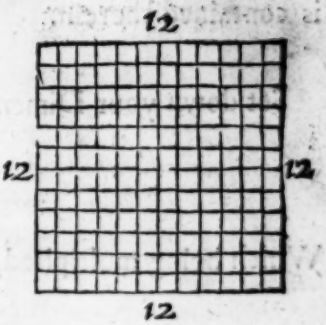
So that the Product of 11 Inches, multiplyed by 9 Inches, which is 99 Inches, contains 8 such Inches, whereof 12 make a Superficial Foot, that is to say, it contains 8 Inches, whereof each Inch is one Inch in breadth, and 12 Inches long; For 8 times 12 is 96, which is $\frac{2}{3}$ parts of a Foot; then the 3 Inches which are remaining, (for the Product was 99 Inches, take 66 from 69, and there remains 3) these three Inches, I say, are $\frac{3}{12}$, or, as I call them, 3 parts of such an Inch, as aforesaid; namely, an Inch in breadth, and 12 in length.

For by the way you must note, That in a Superficial Foot, there is contained 144 Inches, which is the Product of 12 Inches multiplyed by 12, thus ———

12	Length.
12	Breadth.
144	Product.

Or

Or as you may ſee in the Geometrical Figure hereunto annexed.



By this Geometrical Figure, you may plainly perceive, that there is in a Square ſuperficies, being 12 Inches on each ſide, 144 Superficial Inches; for every little Square in the Geometrical Figure repreſents an Inch: So conſequently in half a Foot there is 72 Inches, being produced from an oblong, being 12 Inches in Length, and 6 Inches in Breadth.

Which oblong you may perceive, or imagine in this ſquare Geometrical Figure, by taking the whole Length one way, and half the Length, or 6 of the little Squares the other way: Therefore I ſhall not need here to deſcribe the Figure of an Oblong.

Alſo half a Superficial Foot, which is 72 Inches, is produced from an Oblong very near a Square, being 9 Inches in Length, and 8 Inches in Breadth, for 9 times 8 is 72.

Likewiſe a Quarter of a Superficial Foot contains 36 Inches, which is one half of 72 Inches, or the Product of 6 Inches multiplyed by 6 Inches, or 9 by 4.

I ſhall now Proceed to Dimenſions in Feet.

Suppoſe there is a Timber Floor (or a Stone Pavement, or a Plaiſtered Ceiling, or a Piece of Wainſcot, or any other Superficies) that is 22 Feet in Length, and 18 Feet in Breadth, and you would

know in Carpenters Worke how many Squares there is contained therein.

Feet

Set down your Dimenſion thus ——— { 22 / 18

176

22

Which being multiplied, the Product is ——— 396

Superficial Feet.

Now to bring theſe Feet into Squares, you muſt divide 396, the Content in Feet by 100 (the Content of a Square whoſe ſide is 10 Feet) and the Quotient is 3 Squares and 96 Feet remaining.

See the example ——— 396 (96 Feet.
100 (3 Squares

Here you ſee by the example, that there is 4 Squares of Flooring wanting 4 Feet; or 3 Squares and three Quarters of a Square and 21 Feet.

If the Dimenſion had been Stone Pavement of Maſonry, then there had needed no Diviſion, for they work by the Foot; ſo there had been 396 Feet of Pavement with broad Stone, at ſo much per Foot.

But if it had been Plaiſtering, or Painting, or Wainſcotting, or Hangings, or ſuch like; then you muſt have brought the Feet into Yards, which is done by dividing 396 Feet by 99 Feet, which is the number of Feet contained in a Superficial Yard (or Quadrate) being 3 Feet in Length, and 3 Feet in Breadth.

See the example ——— 3 / 396 / 99 (44 Yards.

Thus having divided 396 Feet by 9 Feet (or one Yard) you have 44 Yards in the Quotient, which is the number of Yards contained in a Superficies of Plaiſtering, or Painting, or Wainſcot, or Hangings, or ſuch like, being 22 Feet in Length, and 18 Feet in Breadth.

Another Example in Feet.

Suppoſe a Floor or Partition be 146 Feet in Length, and 97 Feet in Breadth.

	Feet.
Set down the Dimenſion thus ———	146
	97
	1022
	1314
Being multiplied the Product is ——	14162 Feet

Which being divided by 100, produceth 141 Squares and 62 Feet: Or you need not divide it, but read it thus, one hundred forty one hundred and ſixty two Feet, which is 141 Squares and an half and 12 Feet.

If you were to bring the ſame Dimenſion into Yards, then you muſt divide the Product of the Multiplication being 14162, by 9 Feet (or one Yard) as you did in the foregoing example.

See this example ——— 563 / x4x62 / 9999 (5 Feet. 1573 Yards.

So you have in the Quotient 1573 Yards and 5 Feet remaining, which is half a Yard and half a Foot. Thus much may ſerve for Meaſuring of Dimenſions of whole Feet.

I ſhall now Proceed to Dimenſions in Feet and Inches, which is ſomething more difficult.

Suppoſe a Floor, or Partition, or any other Superficial thing, as Plaiſtering, or Painting, *&c.* be 21 Feet and 6 Inches in Length, and 15 Feet and 6 Inches in Breadth; and it is required to know how many Superficial Feet there is contained therein, and conſequently how many Squares or Yards.

Set down your Dimenſion thus ——

	Feet	In.
	21	06
×	15	06
	105	
	21	
	315	

Then multiply the Feet, and the Product is 315

Then for the 6 Inches in Length, and 6 in Breadth, multiply them Diagonal or Croſs-wiſe into the Feet, ſaying 15 times 6 Inches (or 6 times 15 Inches) is 7 Feet and 6 Inches: Or thus more briefly, (6 Inches being half a Foot) ſay, the half of 15 Feet is 7 Feet and 6 Inches, which you muſt add to the former Product 315. This being done, in the next place you muſt multiply the 6 Inches in the Breadth into the 21 Feet in the Length; ſaying, the half of 21 Feet is 10 Feet and 6 Inches, which you muſt likewiſe add to the reſt.

Then

Then laſt of all, you muſt multiply the Inches of the Breadth into the Inches of the Length; ſaying, 6 times 6 is 36, which 36 is accounted but for 3 Inches, the reaſon whereof, in the next following Page but one ſhall be ſhewn

For note, How many times 12 you have in the Product of the Inches, being multiplied in themſelves, ſo many Inches you muſt add to the former work, as in this example, you have 3 times 12 in 36. As you may ſee in the following example.

Whence note, That it is uſual to begin the Multiplication towards the left hand firſt, namely the Feet into the Feet, and afterwards the Inches into the Feet, *&c.*

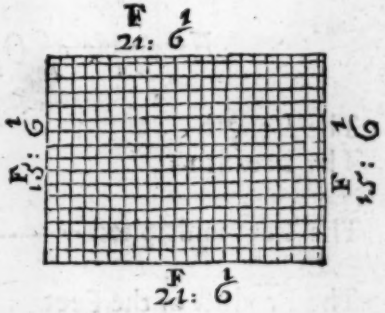
Example of Operation.

	Feet	In.
The Length	21	6
The Breadth	15	6
The Feet multiplied	105	
	21	
The Product of the Feet	315	
The half of 15 Feet, or the Product of 6 Inches, being multiplied by 15 Feet —	7	6
The half of 21 Feet, or the Product of 21 Feet, being multiplied by 6 Inches—	10	6
The Product of 6 Inches multiplied by 6 Inches, which produces 36, or 3 times 12 which as you read before is 3 Inches —		3
	333	3

Then add your Summs together, beginning at the Inches on the right hand, ſaying, 3 and 6 is 9, and 6 is 15 Inches, which is, 1 Foot and 3 Inches,

then ſet down 3 under the Column of Inches, and carry 1 to the Feet, and ſay 1 that I carry and 7 makes 8, and 5 makes 13; then ſet down 3 in the place of Units, under Feet, and carry 1 to the next place, ſaying 1 that I carry and 1 is 2, and 1 makes 3, which I ſet down on the left ſide of the former 3 (to wit) under the place of tens. Then I proceed in the next place to 3, which being in the place of hundreds is 300, and ſet that 3 down on the left ſide of the two other.

So the whole Product of 21 Feet and 6 Inches, being multiplyed by 15 Feet and 6 Inches, is 333 Feet and 3 Inches, as you have it above in the example: I think it convenient here to add a Geomeometrical Figure for the better underſtanding of what hath been ſaid.



By this Geometrical Figure, you may perceive the truth of what hath been ſaid.

Firſt, here you ſee that the whole Squares or Feet are 21 in Length, and 15 in Height or Breadth, which produce 315 Feet.

Secondly, In the upper part of the Figure, you ſee 21 half Squares or half Feet, which is, the 6 Inches which you multiplied by the 21 Feet, the Product whereof was 10 Feet and an half; ſo in the Figure, accounting two half Squares (or half Feet) for one whole one, you will find 10 Feet and an half, as before by Arithmetick.

Thirdly,

Thirdly, On the right ſide of the Figure, you ſee 15 Oblongs or half Feet, which are the ſix Inches that you multiplied by the 15 Feet, which did produce 7 Feet and an half; ſo in the Figure, accounting two halfs for one whole, you will have 7 Feet and an half, as you had before by Arithmetick.

Fourthly, In the uppermoſt Angle on the right ſide of the Figure, you ſee a little Square which is 6 Inches in Length, and 6 Inches in Breadth. This is the 6 Inches that you multiplied by 6 Inches, which produced 36 Inches, for which there was ſet down but 3 Inches: The reaſon whereof is this.

Theſe 3 Inches are the $\frac{1}{4}$ of 12 Inches, ſo is 36, the $\frac{1}{4}$ of 144 Inches, of which I have declared before, that there is 144 ſquare Superficial Inches in a Superficial Foot.

Therefore for conveniency and brevity in adding ſeveral Dimenſions together, the odd Inches being multiplied in themſelves, 144 Inches are accounted as 12 Inches, and 12 of 144 are accounted as one Inch.

Now if you will give your ſelf the trouble to tell the Squares in the Geometrical Figure, which Squares repreſent Feet, and the half Squares or Oblongs which repreſent half Feet, and add them together, you will find that they make 333 Feet beſides the little Square in the upper Angle of the Figure on the right ſide, which repreſents 3 Inches, or $\frac{1}{4}$ of a Foot; ſo you will have 333 Feet and 3 Inches, as was produced by Arithmetick before.

By what hath been declared, I think it eaſie for a mean capacity to underſtand how to caſt up the foregoing Dimenſion, and bring it into Feet.

Now

Now to know how many Squares of Carpenters Work there are in this Dimension, whose Product is 333 Feet and 3 Inches; you must either divide 333 by 100, or else cut off the two last Figures with a stroak thus 3|33 and read it thus, 3 Squares and 33 Feet and 3 Inches.

So in 14162 Feet, cut off the two last Figures thus 141|62, and then you read it 141 Squares and 62 Feet.

So in other Numbers consisting of three Figures or more, cut off the two last Figures to the right hand, and the remaining Figures will be Squares.

But if you would know how many Yards there is in 21 Feet and 6 Inches in Length, and 15 Feet and 6 Inches in Breadth; you must as before is taught, divide the Product of the Dimension being 333 Feet, by 9, being the number of Feet in a Square Yard Superficial,

As in example $\begin{array}{l} \cancel{6} \\ \cancel{333}\,(37 \text{ Yards} \\ \cancel{99} \end{array}$

So your Quotient is 37 Yards; the odd 3 Inches must be added after you have divided.

Take another Example.

Suppose you have a Floor or Partition, that is 35 Feet and 10 Inches in Breadth, and 46 Feet 9 Inches in length. Set down your Dimension as before, and multiply the whole Feet 46, by the whole Feet 35; and the work will stand thus,

Feet	In.
46	09
35	10
230	
138	

Secondly, Multiply the Inches into the Feet cross-ways, saying 9 times 35

is

is 315, which must be set a part by it self, as you may see in the example in this Page.

Thirdly, You must multiply the 10 Inches by the 46 Feet, which produces 460, this you must add to the 315, which before you set apart, and they make 775 Inches, which you must divide by 12, to bring the Inches into Feet, which being divided, the Quotient is 64 Feet, and 7 Inches remaining, which 64 you must add to the multiplication of the Feet, and the 7 to the Inches.

See the example of the Inches multiplied into the Feet.

46 X 09
35 X 10

The Product of the 10 Inches being multiplied into 46 Feet is 460 Inches

The Product of 9 Inches multiplied by 35 Feet is 315 Inches

Which being added is 775 Inches

The 775 Inches being divided by 12, produces as in the Margin 64 Feet and 7 Inches, which must be added to the multiplication of the Feet in the preceeding Page, and then the work will stand thus

x
x5(7
775(64
xxx
x

46 X 09
35 X 10

Fourthly, You must multiply the 10 Inches belonging to the Breadth, by the 9 Inches belonging to the Length, and the Product will be 90 Inches of 144 Inches; or 7 Inches and an half of 12 Inches, for if you divide 90 by 12, you

230
138
64 7
7

will

will have 7 in the Quotient, and 6 remaining, which 7 in the Quotient is 7 Inches, and must be added to the place of Inches; and the 6 that remains over and above the Quotient, being the $\frac{1}{2}$ of 12 is half an Inch, which is seldom accounted in Carpenters Work: But although it is not usually done, yet if you are minded to be exact in Measuring, you may set down these odd parts of Inches by themselves, and at last add them all together, and for every 12 of these parts you may add 1 Inch, and for 144 of them you must add 1 Foot, as you read before.

Then adding all your Summs together, you find that 46 Feet 9 Inches, multiplied by 35 Feet 10 Inches, is 1675 Feet 2 Inches.

Which if you are to bring into Squares, you cut off the two Figures next the right hand thus 16|75 and then you read it 16 Squares and 75 Feet (or 16 Squares and $\frac{1}{4}$ of a Square.)

An Example of the Whole Work.

	Feet	In.
The Length.	46	09
The Breadth.	35	10
The Product of 46 by 5.	230	
The Product of 46 by 3.	138	
The Product of all the Inches multiplied by all the Feet.	64	07
The Product of the 10 Inches multiplied by 9.		07
The whole Product.	16|75	02

Also by the way take notice that Girders Ends, and Ends

Ends of Breſt-Sommers, and Plates, and ſuch like, muſt be remembred and ſet down in your Bill of Meaſurement.

Alſo you muſt allow in your Meaſure, for the Ends of the Joiſts that lye in the Walls; if it be Flooring unboarded that you are to Meaſure, then you ought to allow 9 Inches into each Wall that way the Joiſt Ends are laid.

But if you Meaſure Flooring and Boarding, then if you allow 6 Inches it is reaſonable, becauſe though the Timbers go into the Walls 9 Inches, yet the Boarding goeth but home to the Wall.

And as you add theſe things you muſt alſo remember to deduct the Stairs and Chimneys where the Workman findeth materials, elſe not.

Thus much will ſerve as to the Meaſuring of Flooring, Partitioning and ſuch like; in the next place I ſhall treat,

Of the Meaſuring of Roofs.

Suppoſe a Building to be 30 Feet in Breadth, from the out-ſide of one Wall to the out-ſide of the other, and 65 Feet in Length from Out to Out of the Walls, and you would know how many Squares of Roofing there is in the Roof of this Building.

If the Roof be true Pitch, you need not Meaſure the Length of the Rafter, but take this for a general Rule.

That the Length of the Raſter is $\frac{1}{4}$ of the Breadth of the Building, therefore the Breadth being 30 Feet, the Length of the Rafter will be 22 Feet and 6 Inches, (which is $\frac{1}{4}$ of 30) which being doubled (for ſo it muſt be for both ſides) makes 45 Feet. Then multiply

ply 45 by the Length of the Building being 65, and the Product will be 29|25, which is 29 Squares and a quarter.

But the uſual way which is ſomething briefer is thus, multiply the Length 65 by the Breadth 30, and that produces 1950 Feet for the flat of the Building, then add half the number of Feet in the flat (to wit) 975 to 1950, and the Product is 2925 as before.

You may add the half thus, ſaying the half of 19 is 9, then becauſe twice 9 is but 18, and there remains 1; you muſt carry that 1 to the next Figure being 5, and for that 1 which remained of the 19 after it was halfed, you muſt add 10 to 5, which makes it 15.

Then ſay the half of 15 is 7 and 1 remaining, add that 1 to the Cypher, and it makes 10.

Then ſay the half of 10 is 5.

See both the Examples.

	Feet	In.
The Length of the Rafter on one ſide of the Roof.	22	6
The Length of the Rafter on the other ſide.	22	6
The Length of both Rafters being added.	45	0
The Length of the Building to be multiplied by 45.	65	0
	225	
	270	
The Product.	2925	0

An

An Example of the other way called Measuring on the flat.

	Feet	In.
The Length.	65	0
The Breadth.	30	0
The Product of the Length by the Breadth on the flat.	1950	0
The half flat to be added to the flat.	975	0
The Product of the flat and half.	2925	0

Note, When you Measure Roofing this last way, being called Measuring on the flat, you must write over your Dimensions, *Flat and Half*; or 1 ½ to signify that after you have multiplied the Length by the Breadth, you must add half that Product to it self.

Note, If the Building that you are to Measure be broader at one end than it is at the other, you must measure the Breadth in the middle, both for the Flooring and the Roofing, or else measure each end, and add the Summs together, and take half the Product: As suppose a Building to be 14 Feet wide at one end, and 12 Feet wide at the other, they being added make 26, the half whereof is 13 for the width of the Building.

How to Measure a Gable-end.

Suppose a Timber Building be 40 Feet in Breadth, and you have measured all the Carcass of the Building except the Gable-end, or Ends.

To

To Measure this Gable-end, multiply half the Base by the whole Perpendicular; or half the Perpendicular by the whole Base gives you the Area or content of it.

I call the Plate whereon the Rafters stand, the Base.

The Base being 40 Feet, the Perpendicular is 22 Feet and 4 Inches.

See the Example.

	Feet	In.
The half of the Base.	20	0
The whole Perpendicular.	22	4
The Product of 2 multiplied by 20.	40	0
The Product of 20 by 20.	400	0
The Product of the 4 Inches multiplied by the 20 Feet.	6	8
The total Product.	446	8

The other Example.

	Fe:	In:
The whole Base.	40	0
The half of the Perpendicular.	11	2
The Product of 40 by 1.	40	0
The Product of 40 by 10.	40	0
The Product of the 2 Inches by the 40 Feet	6	8
The total Product.	446	8

Thus you see both ways agree, and the Gable-end contains 4 Squares and 46 Feet and 8 Inches.

If

If there be two Gable-ends equal, or both alike in the building, then you muſt ſet down the total Product twice.

If there be four Gable-ends, and each of them of one bigneſs, then you muſt ſet down the total Product four times.

Note, If the Building be Square (*viz.*) hath four right Angles, and the Breadth 40 Feet; and the Roof true Pitch.

The length of the Rafter, and alſo of the Hip Rafter, and the Angles which they make; alſo the length of the Diagonal Line, and of the Perpendicular, are as in the following Table.

As you may ſee by the Figure ABCD hereunto annexed.

		Feet	In.
The Breadth of the Building.		40	00
The Length of the Rafter.		30	00
The Length of the Hip Rafter.		36	00
The Length of the Diagonal Line.		56	$06\frac{2}{3}$
The Length of the Perpendicular.		22	$04\frac{1}{4}$
		Deg.	Min.
Rafter Angles	At Foot	48	10
	At Top	41	50
Hip Angles	At Foot	38	22
	At Top	51	38

Explanation of the Fig. ABCD.

AB, The Breadth of the Building.
AP, The half Breadth.
AE, or EB, The Length of the Rafter.
EP, The Perpendicular.

AF, or FB, The Length of the Hip Rafter.

AGH, The Angle that the Hip makes at Top, and K*i* the measure of it.

HAG, The Angle that the Hip makes at the Foot, and LI the measure of it.

PAE, The Angle that the Rafter makes at Foot, and P*a* the measure of it.

AEP, The Angle that the Rafter makes at Top, and *a*, *n*, the measure of it

LIP the Quadrant of a Circle divided into 90 Degrees for the measuring of the Angles.

AC, The Diagonal.

LM, The Line over which the Rafters must be placed, that the upper ends of the Hip Rafters are fixed to.

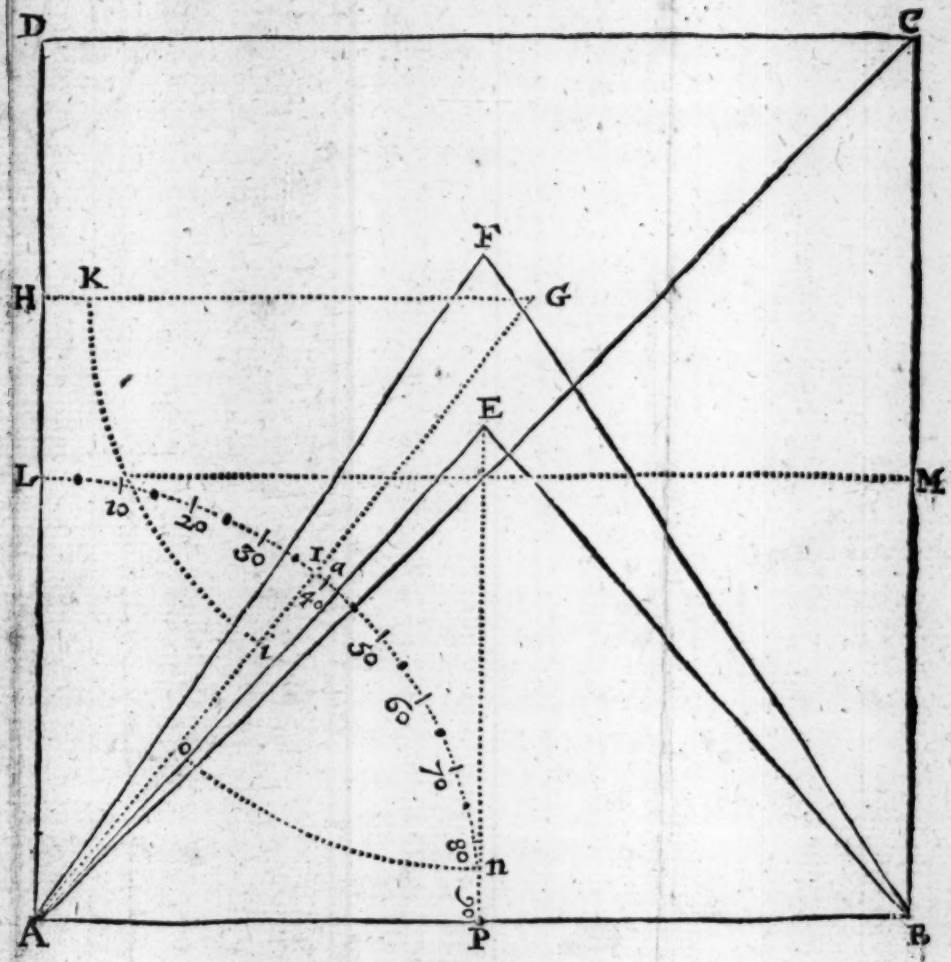
The Length of the Hip Rafter is found by taking the Length of the Rafter AE, and setting it on the Perpendicular from P to F, draw FA the Hip Rafter.

If you have a Roof of any other width, you may describe it from this Figure, or you may find the Length of the Hip Rafters, and the Length of the Diagonal, and likewise of the Perpendicular by the Rule of Three, and the foregoing Table.

The Angles are always equal to those in the Table, let the width be what it will, provided the Building be right Angled, and the Roof the usual true pitch, (that is, the Rafters Length to be $\frac{3}{4}$ of the Breadth of the Building.)

To Measure a Hipt Roof.

Suppose a Building to be 60 Feet in Length, and 40 Feet in Breadth, standing alone, being Hipt from each



Place this between Page 208 and 209.

each Angle, the Angles being right, and the Roof true Pitch, and it is required to know how many Squares of Roofing there is in it.

Multiply the whole Length being 60 Feet, by 30 Feet the Length of the Rafter, for one ſide of the Roof.

Or add 30 and 30 together, they make 60 for both ſides of the Roof, then multiply 60 by 60, the Product is 3600, or 36 Squares in that Hipt Roof.

Which I thus Demonſtrate.

The Length of the Roof on the Ridge, from Hip to Hip is 20 Feet, and the Length of the Roof at the Plate is 60 Feet.

Add theſe two ſumms together, and they make 80 Feet, whereof take half, that is 40 Feet, which is the mean Length of the Roof, if you meaſure in the middle between the top and bottom: Then multiply this 40 Feet by 60 Feet, being the Length of both Rafters on each ſide the Roof, and the Product is 2400 which is the meaſure of the two ſides of the Roof.

Proceed we in the next place to the two Ends.

Being each of them 40 Feet in Length, and the Rafter of each end 30 Feet in Length; each of the ends being a Triangle, which you muſt meaſure as you did the Gable-end.

Multiply 40 Feet being the Baſe, by 15 Feet being one half of the Rafter (or Perpendicular as you may call it, although it lean from an upright) and the Product is 600.

The other end of the Roof being the ſame as this, add 600 to this 600 it makes 1200, which being added to 2400 the Product of the two ſides, make 3600, or 36 Squares as before, which was to be Demonſtrated.

Suppoſe you were to meaſure the Roofing of a Corner houſe being 60 Feet long, and 40 Feet broad, and joining to other Houſes each way, then there will be but one Hip, and one Sleeper, yet the Dimenſions will be the ſame, and the ſame number of Squares in it, as in the former that was Hipt, provided the Building be Square, and the Roof true pitch.

See the Examples.

	Feet	In.
The whole Length of the Roof.	60	00
The Length of both Rafters.	60	00
The Product or number of the Squares	3600	00

The Demonſtration.

	Feet	In.
The Length of the Roof on the Top is	20	00
To which add the Length of the Roof at the Plate	60	00
The Summ is.	80	00
The Length of both Rafters	60	00
Being multiplied by 40 the mean between 60 and 20	40	00
The Product is	2400	00
Which is the Content of the two ſides.		

Now

Now for the two Ends.

	Feet	In.
The Length of the Base	40	00
Multiplied by the half Length of the Rafter.	15	00
	200	00
	40	
Produces	600	00
To which add the measurement of the other end.	600	00
The summ is the Content of both the ends,	1200	00
To which add the measure of both sides	2400	00
The summ is the Area or Content of the whole Roof, as before	3600	00

How to find the Length of the Hip for a Roof that is 20 Feet in Breadth, by the Rule of Three.

You see by the foregoing Figure A B C D, that a Roof being 40 Feet in Width or Breadth, gives a Hip Rafter that is 36 Feet in Length; therefore state your Question thus. If 40 give 36, what will 20 give,

Multiply and Divide, and you will find 18

36
20
720

3
720 (18
40
4

After the same manner you may find any of the rest. You must remember to add the Rafters Feet, and Eves Boards, in the Bill of Measurement.

CHAP. IV.

Measuring of Glaziers Work.

THe Measuring of Glaziers Work is the same as of Carpenters Work; only Carpenters Work is not measured any nearer than two whole Inches.

But Glaziers Work is sometimes measured to $\frac{1}{4}$ of an Inch; and the best and readiest way to measure Glazing, is to take the Dimensions with a sliding Ruler, such as Glaziers generally use, which Ruler is divided Decimally, a Foot into 100 parts, so that a whole Foot contains 100 Parts or Divisions; $\frac{3}{4}$ of a Foot contains 75 Parts; $\frac{1}{2}$ of a Foot contains 50 Parts, and $\frac{1}{4}$ of a Foot contains 25 parts.

Those that desire to measure the Decimal way, I referr them to a Treatise lately written by my worthy Friend *Tho. Hammond* (Entituled *A new and exact way of Mensuration*) and indeed it is a very ready way of Measuring, provided our two Foot Rules were divided into 20 equal Parts, or a Foot Rule into 10 Parts, (or as I may call them Inches) and every one of those Parts divided into 10 equal parts.

But to come to the thing intended (to wit) to measure Glazing by Vulgar Arithmetick, and with a two Foot Ruler divided into 24 Parts, and each of those into 8 Parts, as they are commonly made and used.

Suppose you have a Pane of Glass which is 5 Feet 8 Inches and an half in Length, and 5 Feet 7 Inches and $\frac{1}{2}$ in Breadth, and you would know the Area or Content thereof.

Set

Set down your Dimenſion thus.

	Feet	In.	Parts.
}	5	8	6
}	5	7	3

And read it, 5 Feet, 8 Inches and 6 Parts of 12, which is the ½ Inch, by 5 Feet 7 Inches and 3 parts of 12, which is the ¼ Inch.

Then multiply the Feet and Inches one by the other, as you are taught in the foregoing Pages; ſaying 5 times 5 is 25 Feet, which being ſet down under the Feet; multiply croſs ways 5 by 8, which is 40 Inches, or 3 Feet and 4 Inches, this being ſet down.

In the next place multiply 7 by 5, which is 35 Inches, or 2 Feet and 11 Inches, this being ſet down.

In the next place multiply the 7 Inches by the 8, it is 56 Inches, of which 144 make a Superficial Foot, for which you muſt ſet down 4 Inches, becauſe you can have but 4 times 12 out of 56, and there remains 8: Therefore I ſet down 4 Inches and 8 Parts, of which Parts 12 make an Inch, 12 of which Inches are accounted a Superficial Foot.

In the next place multiply the parts of Inches croſs-ways; firſt into the Feet, ſaying 6 times 5 makes 30, which is 2 Inches and 6 parts, this being ſet down.

Say 5 times 3 is 15, or 1 Inch and 3 parts, which you muſt put alſo under the reſt.

Then multiply the parts of Inches croſs ways into the Inches, ſaying 6 times 7 is 42, which is 3 parts and an half, for which I ſet down 3 under the place of parts of Inches, and 6 againſt it towards the right hand, which ſignifies ½ a part of a part; (for 12 of theſe parts make one ſuch part, whereof there are 12 in an Inch) Line meaſure, this being done, multiply 3 by 8

(which is 3 parts by 8 Inches) that makes 24 parts, for which you must set down 2 under the place of parts of Inches.

Note, That as in multiplying the Inches by the Inches, you account every 12 of the Product for 1 Inch, so in multiplying the parts of Inches into the Inches, you must account every 12 of the Product for 1 part, as in multiplying 3 parts by 8 Inches produceth 24, which is twice 12, therefore I set down 2 under the place of parts.

Lastly, you must multiply the parts of Inches by the parts (if you will measure so near) saying 3 times 6 is 18, which is 1 part and an half of a twelfth part of an Inch Line measure; see the Dimension cast up.

Example.

	Feet	In.	Parts.
	05	8	6
	05	7	3
The Product of 5 Feet by 5 Feet is	25	0	0
The Product of 8 Inches by 5 Feet is	03	4	0
The Product of 7 Inches by 5 Feet is	02	11	0
The Product of 7 Inches by 8 Inch. is	00	4	8
The Product of 6 Parts by 5 Feet is	00	2	6
The Product of 3 Parts by 5 Feet is	00	1	3 Pa.
The Product of 6 Parts by 7 Inches is	00	0	3 6
The Product of 3 Parts by 8 Inches is	00	0	2 0
The Product of 3 Parts by 6 Parts is	00	0	0 1 $\frac{1}{2}$
The total Product is	31	11	10 7 $\frac{1}{2}$

Thus

Thus you ſee that the Area or Content of 5 Feet 8 Inches and an half (or 6 parts of 12) in Length, being multiplied by 5 Feet 7 Inches and $\frac{1}{4}$ (or 3 parts of 12) in Breadth; is 31 Feet, 11 Inches and 10 Parts of an Inch, accounted to have 12 Parts, and 7 and $\frac{1}{2}$ of one of thoſe Parts; but as for the laſt 7 Parts and $\frac{1}{2}$, they are not worth the ſetting down nor taking notice of; the value of thoſe parts being ſo ſmall.

The truth of this Area or Content may be proved three ſeveral ways.

Firſt Decimally, Secondly Geometrically, Thirdly by Vulgar Arithmetick.

Firſt Decimally thus; the Decimal of 8 Inches and $\frac{1}{2}$, is 71, and the Decimal of 7 Inches and $\frac{1}{4}$ is 60 and ſome ſmall matter more: Therefore in Decimals, I ſet down the Dimenſion thus.

	F. P.
{	571
{	560
	34260
	2855
	31|97|60

And having multiplied it, I find the Product to be 319760, then cut off the 2 Figures next to the left hand with a ſtroak as you ſee in the example, and they repreſent 31 Feet.

Then the remainder is 9760 of 10000, and by cutting of two Figures more thus 97|60 it is 97 of 100 and ſomething more, it is near 98 parts of 100, 92 of which parts, is the Decimal of 11 Inches, and the 6 parts which remain are $\frac{6}{10}$ of an Inch; ſo that by this you ſee the Decimal way of meaſuring and this Vulgar doth agree.

Secondly, you may prove it Geometrically, if you deſcribe a Geometrical Figure (after the ſame manner that the Figure is deſcribed in Pag. 198 of *Carpenters Work*, being drawn according to a Scale;) 5 Feet 8 Inches

8 Inches and $\frac{1}{2}$ in Length, and 5 Feet 7 Inches and $\frac{1}{4}$ in breadth; and then divide the Feet and Inches, and parts; and you will find it to agree with the Area or Content before produced.

Thirdly, it may also be proved by Vulgar Arithmetick thus.

Reduce the Feet and Inches into the least Denomination (to wit) Quarters of Inches, and then multiply them one by another.

To do this, multiply 68 Inches (which are the Inches in 5 Feet and 8 Inches) by 4, and that brings them into Quarters of Inches Line measure, then add 2 to the Product, for the half Inch in the Length.

68
4
272
2

Thus in the Length you have 274 Quarters of Inches

The Breadth being 5 Feet 7 Inches and $\frac{1}{4}$; multiply 67 (the Inches in 5 Feet and 7 Inches Line measure) by 4, being the Quarters in an Inch, Line measure, and to the Product add 1, which is the $\frac{1}{4}$ Inch in the Breadth.

67
4
268
1

And in the Breadth is contained 269 Quarters of Inch.

Thus

Thus having brought the Length and Breadth into the leaſt Denomination (to wit, Quarters of Inches) I ſet down my Dimenſion thus and multiply it

	274	
	269	
	2466	
	1644	
	548	
The Product is	73706	Quarters.

Of which Quarters there are 16 in an Inch Superficial meaſure (to wit, 4 in Length, and 4 in Breadth) and of which Inches there are 144 in a Foot Superficial meaſure (to wit, 12 in Length, and 12 in Breadth.) Therefore I multiply 144, the Inches in a Superficial Foot, by 16, the Quarters in a Superficial Inch, and

	144	
	16	
	864	
	144	
In a Superficial Foot there is	2304	Quarters.

Then Divide 73706 the Superficial Quarters of Inches in the whole Dimenſion, by 2304 the Superficial Quarters of Inches in a Foot.

228
~~04582~~
~~73706~~ (31 Feet.
~~23044~~
~~230~~

And the Quotient is 31 Feet, and the remainder is 2282 Quarters, which are not a Foot.

Therefore divide 2282 by 192 (which is the number of Quarters contained in 12 Inches in Length, and 1 Inch in Breadth, which 192 is the Product of 12 by 16.)

12	17
16	~~360~~
72	~~2282~~ (11 Inches.
12	~~1922~~
192 Quar.	~~19~~

And the Quotient is 11 Inches, and 170 the remainder, which is 170 Parts or Quarters, of an Inch in Breadth and 12 Inches in Length, which Inch as I told you before contains 192 Quarters.

Wherefore divide this 170 by 16 (to wit, the Quarters that are contained in an Inch in Length, and an Inch in Breadth.)

1
~~170~~ (10 Parts.
~~166~~
~~1~~

And the Quotient is 10, and the remainder 10.

Thus you see the Product of 5 Feet, 8 Inches, and $\frac{1}{2}$ in Length, being multiplied by 5 Feet, 7 Inches, and

and $\frac{1}{4}$ in Breadth, is 31 Feet, 11 Inches, 10 Parts and $\frac{10}{12}$ of a Part, which $\frac{10}{12}$ of a Part is in value the ſame that 7 and $\frac{1}{2}$ is, of a Part containing 12, as you found it by the firſt way of working in Vulgar Arithmetick.

Another Example.

Suppoſe a Pane of Glaſs be 5 Feet, 3 Inches, and $\frac{1}{2}$ long, and 2 Feet, 4 Inches and $\frac{1}{2}$ broad, and you deſire to know the content thereof.

	Feet	In.	Parts.
Set down your Dimenſion thus	5	3	6
	2	4	6

Then Multiply the Feet and Inches one into another, ſaying 5 times 2 is 10 Feet, ſet that down under the Feet; then multiply croſs wiſe, ſaying 3 times 2 is 6 Inches, ſet that under the Inches;

Then ſay 4 times 5 is 20 Inches, or 1 Foot and 8 Inches, ſet the 1 under the Feet, and the 8 under the Inches.

Then ſay 4 times 3 is 12, which is 1 Inch.

Next, Multiply the Parts croſs ways into the Feet, ſaying 6 times 2 is 12, which is 1 Inch, which muſt be ſet under the place of Inches.

Then Multiply the 6 Parts by 5 Feet, it makes 30 Parts, which is 2 Inches and 6 Parts, which being ſet down.

Multiply the Parts into the Inches, ſaying 6 times 3 is 18, which is 1 part and an half; for which I put 1 under the place of Parts, and 6 againſt it towards the right hand, which ſignifies $\frac{1}{2}$ or $\frac{6}{12}$ of a Part.

Then

Then Multiply the 6 Parts by the 4 Inches, it makes 24, which is two Parts, therefore add 2 under the place of Parts.

Laſtly Multiply the Parts by the Parts, ſaying 6 times 6 is 36, which is $\frac{1}{12}$ of a Part, ſet this 3 under the 6 on the right hand.

Then adding them all together, the content is 12 Feet, 6 Inches, 9 Parts, and $\frac{2}{12}$ of a Part; which $\frac{2}{12}$ you need not ſet down as you read before, they ſignifying very little; then it will be 12 Feet, 6 Inches, and 9 Parts or $\frac{3}{4}$ of an Inch.

See the whole work.

	Feet	In.	Parts.
The Length	05	3	6
The Breadth	02	4	6
The Product of 5 Feet by 2 Feet is	10	0	0
The Product of 3 Inches by 2 Feet is	00	6	0
The Product of 4 Inches by 5 Feet is	01	8	0
The Product of 4 Inches by 3 Inches is	00	1	0
The Product of 6 Parts by 2 Feet is	00	1	0
The Product of 6 Parts by 5 Feet is	00	2	6 Pa.
The Product of 6 Parts by 3 Inches is	00	0	1 6
The Product of 6 Parts by 4 Inches is	00	0	2
The Product of 6 Parts by 6 Parts is	00	0	0 3
The Total is	12	6	9 9

Note, That in Meaſuring of Glazing, many times in a Building there are ſeveral Window frames of one bigneſs, and in one Window Frame there are ſeveral Panes of Glaſs of one bigneſs or Dimenſion; as in a

ſix

ſix Light Window there are 6 Panes of Glaſs, 3 of them of one Dimenſion, and 3 of another; ſo in a four Light Window there are 4 Panes of Glaſs, 2 of one bigneſs, and two of another.

In a ſix Light Window you need meaſure but one Pane of a ſort, and ſet down each Dimenſion; and becauſe there are 3 of a ſort of one bigneſs, ſet down 3 againſt your Dimenſion, which ſignifies 3 times that Dimenſion.

If it be a four Light Window, and have 2 Panes of Glaſs of one bigneſs, and two of another, then ſet down 2 againſt your Dimenſion.

Example.

If you were to meaſure a ſix Light Window, and the upper Panes were each of them 2 Feet 6 Inches in height (or Length) and 1 Foot 2 Inches in Breadth. And the lower Panes 6 Feet 4 Inches in Length, and 1 Foot 2 Inches in Breadth: You may make but two Dimenſions of theſe 6 Panes of Glaſs.

Setting them down thus

Feet.	In.	Parts.
2	6	0 (3)
1	2	0

Feet	In.	Parts.
6	4	0 (3)
1	2	0

The 3 in the Circle thus (3) ſignifies 3 times the Dimenſion that it ſtands againſt; therefore when you have caſt up the Dimenſion, the Product thereof you

you must multiply by 3, and set that down for the 3 Panes.

Example.

	Feet	In.	Parts.
	2	6	0 (3)
	1	2	0
	2	0	0
		6	0
		4	
		1	
The Product of of one Pane is	2	11	0
Which multiply by 3, because there are 3 Panes of that bigness.		3	
And the Product is	8	9	0

Being the Content of the 3 Panes.

Then the other 3 Panes, being each of them 6 Feet, 4 Inches in Length, and 1 Foot 2 Inches in Breadth.

	Feet	In.	Parts.
Set down thus	6	4	0 (3)
And multiply,	1	2	0
The Product of 6 Feet by 1 is	6	0	0
The Product of 2 Inches by 6 Feet is	1	0	0
The Product of 4 Inches by 1 Foot is	0	4	0
The Product of 2 Inches by 4 Inches is	0	0	8
The total Product or content is	7	4	8
Which multiply by			3

Saying

	7	4	8
			3
Saying 3 times 8 is 24 Parts or 2 Inches, which carry to the Inches, and ſet a Cypher under the Parts, thus			0
Then ſay 3 times 4 Inches is 12 Inches, and 2 that I carry is 14 Inches, that is, 1 Foot and 2 In. ſet down the 2 In. and carry the 1 Foot to the Feet		2	
Then ſay 3 times 7 is 21 Feet, and 1 that I carry is 22 Feet; ſet that down	22	0	0
Add them, and the Product is	22	2	0

For the Area or Content of the 3 Panes.

	Feet	In.	Parts.
The Content of the firſt 3 Panes is	08	9	0
The Content of the laſt 3 Panes is	22	2	0
Being added together is	30	11	0

Which is the Content of all the ſix Panes in the ſix light Window.

Then, ſo many ſix light Windows of the ſame bigneſs as you have in the Building to meaſure, multiply the number of them by 30 Feet and 11 Inches, and the Product is the Content of them all.

To meaſure a four light Window.

A four light Window having 2 Panes of one bigneſs, and 2 Panes of another; you may ſet down but 1 Pane of each bigneſs, and (2) againſt it thus.

Suppoſe the upper Panes to be 2 Feet 1 Inch in Length, and 1 Foot 8 Inches and ½ in Breadth, a piece.

And the lower Panes to be 4 Feet in Length, and 1 Foot 8 Inches and ½ in Breadth a piece

Begin with the two upper Panes. And ſet them down thus

Feet	In.	Parts.	
2	1	0	(2)
1	8	6	
2	0	0	
1	4	0	
	1		
	0	8	
	1	6	
3	7	2	
3	7	2	
7	2	4	

The Content of 1 Pane is
The other Pane being the ſame, add
The Content of both the upper Panes is

Secondly, ſet down the Dimenſion of the two lower Panes thus,

Feet	In.	Parts.	
4	0	0	(2)
1	8	6	
4	0	0	
2	8	0	
	2		
6	10	0	
6	10	0	
13	8	0	
07	2	4	
20	10	4	

The Content of one Pane is
And becauſe there is (2) ſtands againſt the Dimenſion, you muſt multiply it by 2, or elſe adde
The Content of both Panes is
To which add the Content of the upper Panes
And the Content of the 4 Panes in the 4 light Window is

How

How to ſet down Dimenſions in your Pocket Books.

Note, Before you begin to ſet down your Dimenſions, it is convenient to divide the Breadth of your Page or Leaf into ſo many ſeveral Columns as you think convenient, with Lines drawn of Ink: The Leaves of your Pocket Book being of the Breadth of this Book, you may divide a Leaf into four Parts or Columns.

You muſt likewiſe before you ſet down any Dimenſions, expreſs the Work-maſter and the Work-men's names, alſo the place where, and the Day of the Month, and Date of the Year that you meaſure. Likewiſe if the Work that you are to meaſure, be Glazed with ſquare Glaſs, you muſt write *Squares* above your Dimenſions, and over thoſe Dimenſions which are Glazed with Quarries, you muſt write *Quarries*; that when you come to make the Bill of Meaſurement, you may expreſs them ſeverally, becauſe they are of ſeveral prices.

In the next Page I will ſet down all the Dimenſions which you have been taught to caſt up in Glazing, and ſome others, with the Product to each Dimenſion.

Glazing done by A. B. *for* C. D. *in* Long-Acre, *and measured the* 22th. *of* January, 1680.

Quarries.			Products.			Squares.			Products.		
F.	I.	P.	F.	I.	P.	F.	I.	P.	F.	I.	P.
5	8	6	31	11	10	04	03	00	04	11	06
5	7	3				01	02	00			
5	3	6	12	06	09	02	00	00	03	00	00
2	4	6				01	06	00			
2	6	0 (3)	08	09	00	03	00	00	06	09	00
1	2	0				02	03	00			
6	4	0 (3)	22	02	00	06	00	09	30	05	03
1	2	0				05	00	03			
2	1	0 (2)	07	02	04	01	02	00 (2)	07	00	00
1	8	6				03	00	00			
4	0	0 (2)	13	08	00				52	01	09
1	8	6									
			96	03	11						

Explanation of the Columns.

In the first Column towards the left hand, are the Dimensions (which you have been taught to cast up) of Glazing done with Quarries.

In the second Column you have the Product of each Dimension just against it.

In

In the third Column you have 5 Dimenſions of Glazing done with Square Glaſs.

In the laſt Column you have the Product of each Dimenſion juſt againſt it; and at the bottom you have the total Summ of all the Products in that Column, being 52 01 09

Likewiſe at the bottom of the ſecond Column you have the total Summ of all the Products of the Dimenſions done with Quarries, which is 96 Feet, 03 Inches and 11 Parts. As for the odd Parts, you may leave them out when you make your Bill of Meaſurement.

When you are meaſuring and ſetting down the Dimenſions in your Book, whether it be of Glazing, or any other Trade, you muſt leave every other Column vacant or empty, that ſo having ſet down all your Dimenſions in your Book (which you generally do before you caſt up any) when you caſt them up (which muſt be in another Book or Sheet of Paper) you may enter the Product of each Dimenſion juſt againſt it, as you ſee in the Page before.

If there be another to meaſure againſt you, you muſt after you have done ſetting down all your Dimenſions; compare your Dimenſions together, to ſee if the Dimenſions in both your Books agree.

The reaſon why you ſet down the Product of each Dimenſion juſt againſt it in the next Column towards the right hand, is,

If you meaſure againſt another Meaſurer, and there ſhould be a miſtake in either of your caſtings up of the Dimenſions (as it often happens through ſecurity or

negligence) then one by reading over the Dimensions in his Book, with the Product to each Dimension as he goes on; and the other looking in his own Book, the mistake will soon be found, which must be rectified between you.

Therefore, to be certain in casting up your Dimensions, you ought to cast them up twice, if not three times; (to wit) after you have cast them all over once, begin and cast them over again, and see whether it agrees with your first casting up, if not, then cast them up again.

When you make your Bill of Measurement, you must set your name to it at the lower end of the Bill.

An Example of a Bill.

Glaziers work done by *A*, *B*, for *C*, *D*, in *Longshore*, and measured the 22*th* of *January*, 1680.

	l.	*s.*	*d.*
For 96 Feet and 03 Inches of Glazing done with Quarries at 05 *d.* per Foot.	02	00	01 ¼
For 52 Feet and 1 Inch of Glazing with Squares at 07 *d.* per Foot.	01	10	04 ½
The Summ is	03	10	05 ¼

Measured the Day and Year above written by V. M.

You may sometimes happen to meet with Panes of Glass of various formes; as sometimes the top of a Pane of Glass is concluded with a Semicircle, sometimes

times with a Scheam or Segment of a Circle, ſometimes Triangular, and ſometimes Ovilar, and ſometimes an Oval in Glaſs, or a round Pane.

For the Meaſuring of each of theſe and ſuch like, I refer you to the Meaſuring of Planes in the fifth Book, which doth ſucceed this Treatiſe.

Thus much may ſuffice the Ingenious Practicer, as to the Meaſuring of Glazing.

CHAP. V.

THE Meaſuring of Joiners Work is the ſame that is taught before, only there is this difference; Carpenters Work is brought into Squares, and Glaziers into Feet; and Joiners Work muſt be brought into Yards.

Which to do, after you have caſt up all your Dimenſions, and brought them into Feet, divide the whole Product by 9, and that gives you the number of Yards.

You muſt divide by 9, becauſe 3 Feet in Length, and 3 in Breadth, contain 9 Feet, which is a Superficial Yard.

I will ſuppoſe ſome Dimenſions to be caſt up, whoſe Contents or Products are theſe following.

For it would be needleſs to ſpend time to teach the caſting up of Dimenſions in Feet and Inches again, having ſpoken largely and plainly thereto in the precedent Pages.

Wainſcot.

Feet	Inches.
110	08
304	07
065	06
934	02
041	00
073	01
1529	00

9 (8
1529 (169 Yards and 8 Feet.
999

The Total; which divide by 9 to bring the Feet into Yards, as you ſee above, and in 1529 Feet, you find 169 Yards and 8 Feet.

Which muſt be ſet down in the Bill of Meaſurement at ſo much per Yard.

Then, for Cornices, and Baſe, and Sub-baſe, Joiners do them by the Foot, Line Meaſure, or Running Meaſure, as it is called.

Likewiſe Architrave and Freize, they do by the Foot Line Meaſure.

The Chimney-pieces at ſo much a peece.

CHAP.

CHAP. VI.

PAinters Work is Meaſured like Joiners Work, and brought into Yards, only there is this difference.

When you Meaſure Painting upon Cornices, or Moldings of any ſort, you muſt have a Line, and Girt the round Moldings, and bend the Line into the hollow Moldings. Thus when you are to Meaſure a Room, you begin with your Line at the top of the Cornice, where you may faſten it with a ſmall Nail, and girting the round Moldings, and bending the Line into the Hollows and Angles, bring the Line down to the bottom of the Painting, then meaſure the length of the Line with your Rule, and ſet down that ſumm.

Then begin in one Angle of the Room, and meaſure the length of that ſide whereof you took the height, and ſet down the length under the height which before you ſet down, and that is one Dimenſion.

And if the oppoſite ſide of the Room be like unto that which you have already meaſured and ſet down, and the Room ſquare, you may either ſet down that Dimenſion twice, or elſe ſet a Figure of (2) encloſed againſt the Dimenſion, which ſignifies that Dimenſion is 2 times.

Then after you have ſet down all your Dimenſions, and brought them into Feet and Inches by caſting them up, you muſt divide the whole Product by 9, and that brings the Feet into Yards, as you read before: For you muſt not think to learn to meaſure any one

one of these Trades only, but you must begin at the beginning of mensuration (to wit) of Carpenters Work, and so proceed gradually; and be sure to be perfect in casting up one Dimension before you enter upon another.

Thus proceeding with care and diligence, until you have attained to the measuring of Glazing, you may with ease afterwards measure all the rest of the Works that are Superficial.

Indeed the measuring of Bricklayers Work is something more difficult, because it is reduced to a thickness, and so becomes solid measure.

As for the Painting of Windows, they generally are set down at so much per Light, and Casements at so much a peece.

The next Work to be measured is Plaistering, of which I need say no more than this, that it must be brought into Yards; and where a Sommer or Girder lyes below the Ceiling, it is usually deducted where the Workman finds Materials, but not else.

Also in rendring where materials are found by the Workman, sometimes $\frac{1}{4}$ is deducted for the Quarters, but not where Workmanship only is found, because it might be rendred as soon if there were no Quarters.

The measuring of Masons Work (to wit, flat Paving and such like) is measured the same way as all the rest. There is no need of Division, for Masons Work by the Foot.

I come now to treat of the mensuration of Bricklayers Work.

CHAP.

CHAP. VII.

BRick-work, as you read before, muſt be reduced to a certain or uſual thickneſs, in caſting of it up, for which reaſon it becomes ſolid Meaſure.

Which uſual thickneſs is 14 Inches, or as it is generally called, one Brick and half.

By one Brick and half, is meant the length of a Brick and half the length; or the length of one Brick, together with the breadth of another.

For the breadth of two Bricks with a Joint of Mortar between them, is anſwerable to the length of a Brick, the length whereof ought to be, and generally is 9 Inches, and the breadth 4 Inches and $\frac{1}{4}$, which being added together with a Joint of Mortar makes 14 Inches, the aforeſaid uſual thickneſs.

And as Carpenters Work is brought into Squares, Bricklayers Work muſt be brought into Rods, which Rod contains 272 Feet and 3 Inches, and is produced from 16 Feet and 6 Inches in length, being multiplied by 16 Feet and 6 Inches in height or breadth; or a Quadrate whoſe *ſide* is a Pole or Perch.

When you come to Meaſure the Brick-work of a Building, you will find the Walls thereof to be of ſeveral thickneſſes; ſome thicker than a Brick and half, and ſome thinner, as 1 Brick-walls, or Walls 9 Inches in thickneſs.

Which ſeveral thickneſſes muſt all be brought, or reduced to the uſual thickneſs of 1 Brick and half, which to do, obſerve the following Rules.

1. Set down each thickneſs in a Column by it ſelf, (to wit) ſet down your Dimenſions that are 1 Brick

1 Brick (or 9 Inches) in thickneſs in a Column by themſelves.

Alſo thoſe Dimenſions that are 1 Brick and ½ in thickneſs, muſt be ſet in a Column by themſelves.

Likewiſe thoſe of 2 Bricks in thickneſs by themſelves, and thoſe of 2 Bricks and ½, in a Column by themſelves.

And thoſe of 3 Bricks in a Column by themſelves.

So likewiſe if your Walls be thicker than 3 Bricks, (in length) ſet each thickneſs by it ſelf.

By ſetting the Dimenſions by themſelves, is meant, to ſet them in a Page or Column alone ſeverally, and not to intermix Dimenſions of ſeveral thickneſſes in one Page or Column

Notwithſtanding ſome Meaſurers ſet down their Dimenſions as they take them of ſeveral thickneſſes in one Column; but I do not think it ſo convenient for a Learner, as to ſet them down ſeverally.

2. After you have taken your Dimenſions and caſt them up, and by adding the ſeveral Products together (of one thickneſs) have produced the total Product, then to reduce it to 1 Brick and ½, work thus.

If the Dimenſions that you have caſt up, be 1 Bricks length in thickneſs, that is the breadth of 2 Bricks; then you muſt multiply your total Product by 2, which is the thickneſs of the Wall in the leaſt denomination (to wit) two 4 Inches, or the Breadth of 2 Bricks; and divide that Product by 3, (becauſe there is the breadth of 3 Bricks in a Brick and ½) and the Quotient is the Summ of the 1 Brickwork, reduced to 1 Brick and ½.

Example.

Example.

Suppoſe you have caſt up ſeveral Dimenſions of 1 Bricks length in thickneſs, and the Products (or Contents) of each Dimenſion being added together, produceth a total Product of 4976 Feet of Brickwork, being one Brick thick.

To reduce this to 1 Brick and $\frac{1}{2}$ thick, multiply 4976 by 2 (to wit, the breadth of 2 Bricks, and that brings it into the leaſt thickneſs or denomination, namely 4 Inch work, or Brickwork $\frac{1}{2}$ a Brick thick.)

	4976
	2
So the Product of 4 Inch work is	9952

Which divide by 3 (becauſe there are the breadth of 3 Bricks in 1 Brick and $\frac{1}{2}$) and in the Quotient you have 3317 Feet, of 1 Brick and $\frac{1}{2}$ work and 1 remaining.

	x 1
See the Diviſion	9952 (3317
	3333

The 1 that remains is 1 Foot of 4 Inch work, or 4 Inches of 1 Brick and $\frac{1}{2}$ work being reduced.

For if you divide 12 (which are the Inches in that Foot) by 3 (the number of 4 Inches or $\frac{1}{3}$ Bricks in a Brick and $\frac{1}{2}$) the Quotient will be 4, which is 4 Inches or $\frac{1}{3}$ of a Foot.

So likewiſe your Dimenſions that are thicker than 1 Brick and $\frac{1}{2}$, as ſuppoſe 2 Bricks in length, which

is

is called 2 Brickwork, which 2 Brickwork contains the breadth of 4 Bricks, you must in reducing 2 Brickwork to 1 Brick and $\frac{1}{2}$ work; first multiply the summ by 4, and divide the Product by 3.

Example.

If you have 5845 Feet of 2 Brickwork, multiply it by 4.

See the multiplication	5845
	4
The Product is	23380 Feet

Of 4 Inch work, which you must divide by 3.

	2221
See the Division	23380 (7793
	3333

And the Quotient is 7793 Feet of 1 Brick and $\frac{1}{2}$ work and 1 remaining, which is 1 Foot of 4 Inch work, which being reduced as you were taught in the foregoing Page, is 4 Inches (or $\frac{1}{3}$ of a Foot) of 1 Brick and $\frac{1}{2}$ work. Thus in 5845 Feet of Brickwork being 2 Bricks thick (or the Walls being the length of 2 Bricks in thickness) you have when it is reduced, 7793 Feet and 4 Inches of 1 Brick and $\frac{1}{2}$ work, (the usual thickness.)

If your Dimensions be 2 Bricks and $\frac{1}{2}$ in thickness, you must multiply the total Product by 5, and divide the Product of that multiplication by 3.

If your Dimensions be 3 Bricks in thickness, you may save the labour of dividing and multiplying, because

cauſe a Wall of 3 Bricks thick is juſt ſo much more as a Wall of 1 Brick and ½ thick; therefore if you ſet down the Summ of your Product in 3 Bricks, twice, it will be the ſame, as if you ſhould multiply by 6, and divide the Product by 3, for this reaſon, that 6 is as much more as 3.

In meaſuring of Brickwork you muſt ſet down your Dimenſions with the number of their thickneſſes over them, namely over Dimenſions of 1 Brick in thickneſs ſet 1 B. ſignifying 1 Brick; and over Dimenſions that are 1 Brick and ½ in thickneſs, write 1 ½ B, and over Dimenſions 2 Bricks in thickneſs, write 2 B *&c.*

And caſt up your Dimenſions as you are taught in Meaſuring of Carpenters Work and Glaziers Work.

But in Brick Work as well as in Carpenters work, it is uſual to meaſure no nearer than whole Inches, that is to ſay, ½ Inches and ¼ of Inches, nor ¾ of Inches are not ſet down.

For which parts of Inches an allowance is made, when you come to divide your number of Feet to bring them into Rods.

To bring or Reduce Feet into Rods.

After you have reduced all your ſeveral thickneſſes, and brought them to the uſual thickneſs of a Brick and ½, you muſt divide your whole Product by 272 (being the number of Feet contained in a Rod) to bring or reduce the Feet into Rods.

Taking no notice of the 3 Inches belonging to the 272 Feet, being both contained in a Rod.

For you read before, that in a Rod is contained 272 Feet and 3 Inches, which 3 Inches is allowed to make

make good for the parts of Inches, which is not set down when you set down the Dimensions.

But if you are minded to be exact and curious in measuring, you may set down the odd parts of Inches, and cast them up as you were taught in measuring of Glazing, and divide the total Product by 272 Feet and 3 Inches, but this is seldom or never done in Brickwork.

Folded between Page 240, 241, you have designed the ground plat of a Building. Which together with the instructions that follow, will much assist you in making an Estimate for a Building from a design given.

It will also instruct you how to take your Dimensions, and to set them down in your Book, better than by a multitude of words.

The Explanation of the Design.

This design is the ground plat of a Building, being 25 Feet in the Front, and 40 Feet in the Flank or depth.

The Front and Reer Front Walls are 2 Bricks and ½ thick.

The Flank Walls are 2 Bricks thick, as you may perceive by the Scale annexed to the design.

You may suppose this design to be the ground Floor, having no Cellar beneath it.

And the Story to be 11 Feet and 6 Inches in height from the Foundation to the top of the next Floor.

The Windows are 4 Feet in breadth, and 6 Feet 6 Inches in height or length.

The Door-cases in Front and Reer are 4 Feet wide, and 9 Feet in (length or) height.

The

The Chimneys you ſee ſtand back to back, the Jaums being 2 Bricks thick.

The Encloſure of the Stairs, and the Partition next the Entry, are of Timber.

What hath been ſaid, is ſufficient for Explanation.

I ſhall in the next place inſtruct you how to meaſure the Brickwork of this ſtory in the ground plat, which being well underſtood, you may with eaſe meaſure all the reſt of the Stories over it.

I ſhall begin with the Front.

And becauſe all Walls are (or ſhould be) made with a Baſis, therefore we will ſuppoſe the Front Wall and Reer Front Wall to be 3 Bricks thick 6 Inches high, which is 2 Courſes of Bricks; for 3 Inches is generally allowed for a Courſe (or the thickneſs of a Brick laid in Morter.)

So the firſt Dimenſion will be 25 Feet in length, by 6 Inches in heighth, 3 Bricks thick.

The thickneſſes muſt be always ſet over the Dimenſions, as you will ſee in the next Page.

And becauſe the Front and Reer are both of one length, you need not ſet down the Dimenſion twice, but ſet a Figure 2 encloſed in a Circle thus (2) which ſignifies the Dimenſion to be 2 times.

I think it will be beſt for inſtruction to caſt up the Dimenſions as they are taken (or ſet down) but after you underſtand the way, you muſt ſet down all your Dimenſions firſt, and caſt them up afterwards.

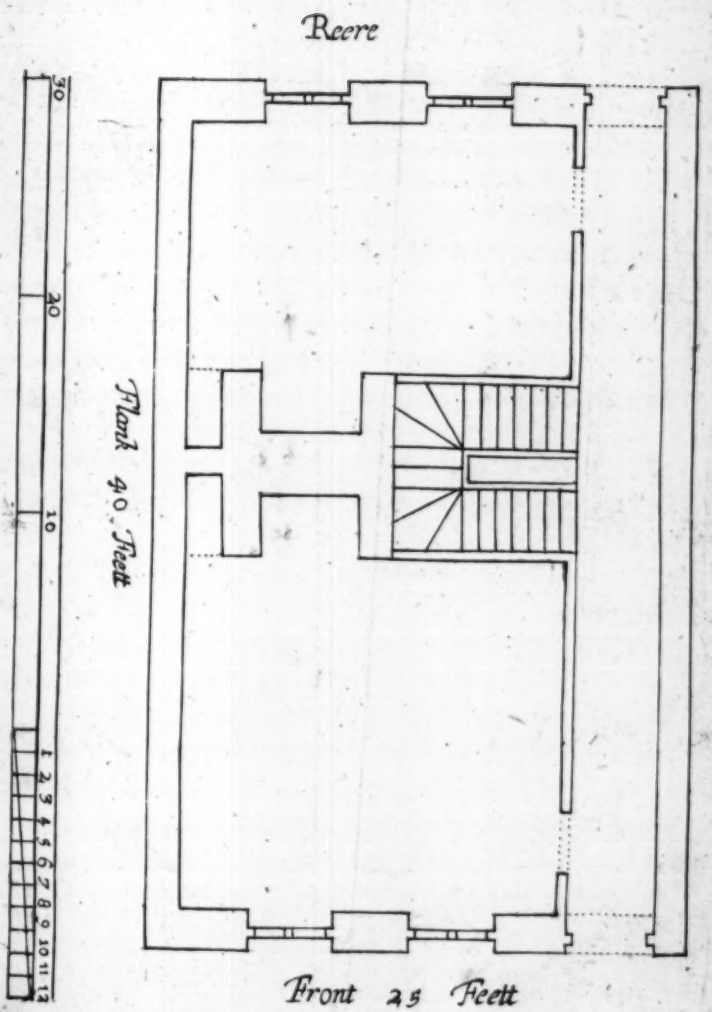
I ſhall alſo ſet the ſeveral thickneſſes of the Dimenſions (of this Story of the ground plat) in one Column, becauſe there will be but a few Dimenſions in this Story, and not above a Dimenſion or two of one thickneſs, and it is not worth while to ſet down a Dimenſion or two in a Column by it ſelf.

Example.

	3 B. Feet	In.	
The length of the Front is	25	00	(2)
The height of the Brickwork is 2 Courses or	00	06	
The Product is	12	06	
The Reer Front being the same Dimenson, and being joined in one Dimension by the Figure (2) you must add	12	06	
Which being added, the Content of the 3 Brickwork is	25	00	
And because 3 Bricks is the double of $1\frac{1}{2}$ B. add	25	00	
	$1\frac{1}{2}$ B.		
And the Content of this first Dimension being reduced into $1\frac{1}{2}$ B. is	50	00	
	$2\frac{1}{2}$ B.		
2ly. I proceed to the remaining part of the Front and Reer, being in height	11	00	(2)
In length	25	00	
Being multiplied	55		
	22		
The Content of the Front is	275	00	
The same being added for the Reer	275	00	
The Content of $2\frac{1}{2}$ B. work in Front and Reer is	550	00	
Which to reduce, you must multiply by	5		
The Product of 4 Inch or $\frac{1}{2}$ B. work is	2750		

Which

Betwen Pag. 240. & 241.



Which muſt be divided by 3 to reduce it to $1\frac{1}{2}$ B. in thickneſs.

22
x750(916
333

And the Quotient is 916 and 2 remaining, which is 916 Feet of $1\frac{1}{2}$ B. and 2 Feet of $\frac{1}{2}$ B. which is 8 Inches of $1\frac{1}{2}$ B. being reduced.

Having done with the Fronts, we proceed to take the Dimenſions of the Flank walls.

And becauſe we meaſured the Fronts from out to out, as we call it in brief, (to wit, from the outſide of one Flank wall, to the outſide of the other) we muſt meaſure the length of the Flank walls within (to wit, from the inſide of the Front wall, to the inſide of the Rear Front wall.) So the thickneſs of the two Front walls, which is 3 Feet and 10 Inches, being taken out of the 40 Feet in depth.

	$2\frac{1}{2}$ B. Feet	Inches.
The length of one Flank wall is	36	02 (2)
The height of the Baſe is	00	06
And becauſe both Walls are alike or equal in length and height, you muſt put (2) againſt the Dimenſion, and it ſerves for both Walls	18	01
	18	01
The Cont. in $2\frac{1}{2}$ B. work of both Walls is	36	02
Being multiplied by	5	
The Product of $\frac{1}{2}$ B. work is	180	10

Being divided by 3 to reduce it to $1\frac{1}{2}$ B

x80(60 1
33 x0(3
3

It is 60 Feet, 3 Inches and $\frac{1}{3}$ of an Inch of $1\frac{1}{2}$ B. work.

	2 B.	
The next Dimenſion of the Flank wall is	36	02 (2)
	11	00
Being multiplied	36	
	361	10
The Content in 2B.work of one wall is	397	10
To which add the other Wall	397	10
The Cont. in 2B.work of both walls is	795	08
Which multiply by		04
The Content of both walls in ½B.work	3182	08

Being reduced to 1½B.the Content of both walls are 1060 Feet and 10 Inches.

3182(1060
3333

Although I caſt up and reduce each dimenſion as I proceed, yet after you underſtand the way, you muſt reduce all your dimenſions of one thickneſs at one time.

Having caſt up and reduced both the Fronts, and both the Flank walls, the next Dimenſions to be taken are the Chimneys: Which are uſually agreed for, at ſo much per Fire-place (or Chimney). Alſo ſometimes they are done by the Rod, at the Rate of the other Work, and then they muſt be meaſured.

Theſe two Chimneys in the deſign, ſtanding back to back, are 5 Feet a piece between the Jaums.

The Jaums are 2 Bricks thick.

The Wall between the Chimneys is 14 Inches thick) to which is added 9 Inches, for a falling back to each Chimney.

Therefore

Therefore we will take the Wall first by it self, which is 9 Feet 10 Inches in length, and 11 Feet 6 Inches the aforesaid height of the Story, and set it down.

	1½ B.	
The height	11	06
The length	09	10
	99	
	4	06
	9	02
		05
The Cont. of the Wall between the Chimneys	113	01

Although heretofore I have put Feet or F. over the place of Feet; and In. or I. over the place of Inches.

Supposing now that you know one place from another, I shall desist doing it, as in the last dimension; and only set the thicknesses over the dimensions.

In the next place suppose the Mantle-tree to lye 5 Feet high, and the falling back to begin at 3 Feet high.

The mean between 5 and 3 is 4, which is the height of the 9 Inch work that is added to each Chimny for faling back, and the length is 5 Feet.

	1 B.		(2)
The Dimension of the falling back of both Chimnys	05	00	
	04	00	
	20	00	
	20	00	
The Content in 1 B.	40	00	

	1 B.
Multiply it by	40 00
	2
The Product is	80

	2 2
Which divide by 3	8∅(26
	3 3

Having reduced it, you find 26 Feet and 8 Inches of $1\frac{1}{2}$ B. work in both fallings back.

The 4 Jaums being 3 Feet 6 Inches deep a piece, above the falling back; being added together make 14 Feet of 2 B. work which must be multiplied by the height of the Story.

For although the Jaums in the next Story be but $1\frac{1}{2}$ B. and from the Mantle-tree in this Story it be wrought but a Brick and half, yet the wings being added, makes two Bricks.

	2 B.
The breadth of the 4 Jaums being added together is	14 00
The height of the Story is	11 06
	14
	147
The Content of the 4 Jaums in 2 B. work is	161 00
Being multiplied by	4
The Content of the 4 Jaums in $\frac{1}{2}$ B. work is	644

	x 2
Which divide by 3 as in the Margin	644 (214
	3 3 3

And

And the Content of the Jaums being reduced to 1½ B. is 214 Feet and ⅔ of a Foot, or 8 Inches.

The 2 breſts of the Chimnys are next to be meaſured, which are 5 Feet a piece between the Jaums, and although they are but 9 Inches thick a piece, yet conſidering the croſs Withs, and the Peers that are wrought from the falling back till the wings gather to them, they are uſually meaſured at the ſame thickneſs that the Jaums are, and at the height of the Story, abating nothing for the vacancy between the Floor and the Mantle-tree.

	2 B.	
The len. of the Breſt between the Jaums is	05	00 } (2)
The height of the Story is	11	06 }
	05	
	52	06
	57	06
The Dimenſion being (2) add	57	06
The Con. of the 2 B. work in both Breſts is	115	00
Which multiply by	4	
Being brought into the leaſt Denomination, or ½ B. it is	460	00

Which divide by 3

xx 1
460 (153
333

Being reduced, the Content is 153 Feet and ⅓ of a Foot (or 4 Inches) of 1½ B.

Thus having taken all the Dimenſions in one Story and caſt them up.

The next thing to be done, is to take the deductions of the Windows and Doors in the Fronts.

Note, That the deductions (to wit, Windows and Doors, and all vacancies whatsoever that you measure) must be set in Columns by themselves, with this note *Ded.* (signifying deductions) over them, together with the thickness of the Wall wherein the Windows or Doors stand that you are to deduct.

Example.

	Ded.	2½ B.	
The height or length of a Window is	06	06	(4)
The breadth is	04	00	

And because there are four Windows in both Fronts, and all of one bigness; therefore put (4) against the Dimension, signifying that that Dimension is 4 times (or that there are 4 Windows of that bigness.)

	Ded.	2½ B.	
The height of a Window	06	06	(4)
The breadth	04	00	
	24	00	
Being multiplied	2		
The Product of 2½ B. work in 1 Win. is	26	00	
Which multiply by	4		and
The Prod. of 2½ B. work in the 4 Win. is	104		
Which multiply by	5		
To bring it into the lest denomination and it is	520		
Which divide by 3; and	21		
Being reduced to 1½ B. there is 173 Feet and 4 Inches.	520 (173		
	333		

There

There are also in Front and Reer 2 Door-cases to be deducted (deduct signifies to take out) of the same thickness of 2½ B.

	Ded.	2½
The height of a Door is	09	007 (2)
The Breadth	04	005
Being multiplied	36	
	2	
The Prod. of 2½ B. work in both Doors is	72	
Which multiply by	5	
Being brought into ½ B. work is	360	

Which divide by 3 as in the margin 360 (120
33
3

Being reduced to 1½ B. there is 120 Feet, as in the Margin.

Note, Though I reduce the deductions here for instruction; hereafter you need not reduce them, but take the Product of the deductions of one thickness out of the Product of Dimensions of the same thickness.

In the following Page I will set down all the Dimensions as they were taken, with the Product of each Dimension in a Column just against it.

And although I said before, that you might divide a Page or Leaf of your Measuring Book into 4 Parts or Colums; yet in Measuring of Bricklayers work, it will be necessary to divide a Page only in two Parts, as you see in the next following Page, that so you may have room to set a name to each Dimension for distinction sake.

Dimensions

Dimensions of the Design			*Products*	
	3 B.		3 B.	
Basis of the Front and Reer	25 00	(2)	25 00	
	06			
	2½ B.		2½ B.	
Front and Reer	25 00	(2)	550 00	
	11 00			
	2½ B.		2½ B.	
Basis of both the Flank Walls	36 02	(2)	36 02	
	06			
	2B.		2B.	
Both the Flank Walls	36 02	(2)	795 08	
	11 00			
	1½ B.		1½ B.	
The Wall between the Chim.	11 06		113 01	
	09 10			
	1B.		1B.	
The falling back of both Chim.	05 00	(2)	040 00	
	04 00			
	2B.		2B.	
The 4 Jaums	14 00		161 00	
	11 06			
	2B.		2B.	
The forepart or brests of both Chimneys	11 06	(2)	115 00	
	05 00			

Having set down all the Dimensions with their Products, in the next place we must set down the deductions of the Windows and Doors with their Products.

Deductions

Deductions in the Design.	*Ded.*	*Products*
	2½B.	
The 4 Windows	06 06 } (4)	2½B.
	04 00 }	104 0
	2½B.	
	09 00 } (2)	2½B.
The 2 Doors	04 00 }	072 0

The next work is to add the Products of each several thickness into one summ.

Products of several thicknesses.

3B.	2½B.	2B.	1½B.	1B.
25 00	550 00	795 08	113 1	40 0
	36 02	161 00		
	586 02	115 00		
		1071 08		

The several Products of each thickness being added.

In the first Column of the left hand, you have 25 Feet of 3 B.

In the second Column you have 586. 2. of 2½B.

In the third Column you have 1071. 8. of 2B.

In the fourth Column you have 113. 1. of 1½B.

In the fifth Column you have 40 Feet of 1B.

The next work must be to take the Products of the deductions out of the Products of the Dimensions.

Products

Products of Ded. in $2\frac{1}{2}$B.

104	0
72	0
The total Product of Ded. in $2\frac{1}{2}$B. is 176	0

Which 176 Feet of $2\frac{1}{2}$B. work being contained in the Windows and Doors, muſt be ſubtracted from 586 Feet and 2 Inches, being the total Product of all the Dimenſions that are $2\frac{1}{2}$B. in thickneſs.

For this Reaſon.

Becauſe when we meaſured the Front and Reer, we meaſured the whole Length and Breadth over the Windows and Doors, allowing no abatement for them.

Note, That whatſoever Windows or Doors, or other vacancies you meaſure over when you take the Dimenſions, you muſt remember to deduct them out of the total Product of the Dimenſions of the ſame thickneſs wherein they are Scituate.

Example.

The doors & windows being in $2\frac{1}{2}$B. work; I ſet down the total Product of all the dimenſions of that thickneſs, which is 586 02

The total Product of all the deductions of that thickneſs which are to be ſubſtracted is 176 00

The Remainder is 410 02

So likewiſe if there had been deductions in the other thickneſſes, you muſt have ſubtracted them before you begin to reduce your ſeveral thickneſſes to the uſual thickneſs of $1\frac{1}{2}$B.

But ſeeing we have no other Ded. in this deſign, to ſubtract out of any other thickneſs, the next work will be to reduce each thickneſs to the uſual thickneſs of $1\frac{1}{2}$B.

Beginning with the greateſt thickneſs (to wit 3B.) we find 25 Feet, which as you read before, becauſe it is juſt as thick again as $1\frac{1}{2}$B. you need not multiply it by 6, and divide it by 3, but add

add just so much more to the Dimension as it is (to wit 25 to 25) and so you will have 50 Feet, which must be set down in a Column with $1\frac{1}{2}$B. over it.

Products reduced

	$1\frac{1}{2}$B.
0050	0

The next thickness to be reduced is $2\frac{1}{2}$B. the total Product whereof is (the deductions being taken out) 410 Feet and 2 Inches, which multiply by 5, and divide the Product by 3.

Example.

	$2\frac{1}{2}$B.	
	410	2
		5
$\frac{1}{2}$B. work	2050	10

~~2~~ ~~1~~ 1
~~2~~ ~~0~~ ~~5~~ ~~0~~ (683
~~3~~ ~~3~~ ~~3~~

The 1 that remains in Division is 4 Inches, and the 10 Inches in the $\frac{1}{2}$B. work, is 3 Inches of $\frac{1}{2}$B. work, which being added is 7 Inches; so the $2\frac{1}{2}$B. work being reduced is 683. 7. which must be set in the Column.

0683	7

	2B.	
The next thickness to be reduced is 2B, the total Product whereof is	1071	8
Which multiply by		4
And the 4 Inch (or $\frac{1}{2}$B.) work is	4286	8

~~2~~ ~~0~~ ~~2~~ 2
~~4~~ ~~2~~ ~~8~~ ~~6~~ (1421
~~3~~ ~~3~~ ~~3~~ ~~3~~

The 2 remaining in Division is 8 Inches, and the 8 Inches of $\frac{1}{2}$B is 2 Inches of $1\frac{1}{2}$B, which added is 10 Inches, so the 2 B. work being reduced is

1428	10

The next thickness being $1\frac{1}{2}$B. needs no reducing because it is the usual thickness, the Product whereof is

0113	1

	1B
The next thickness to be reduced is 1 B. the total Product whereof is	40
Which multiply by	2
And the Product of $\frac{1}{2}$B. work is	80

~~2~~ 2
~~8~~ ~~0~~ (26
~~3~~ ~~3~~

which divide by 3, and the Quotient is 26 Feet and 8 Inches, being reduced;

0026	8
2302	2

The Products of all the thicknesses being reduced is

Having

Having reduced all the thicknesses to $1\frac{1}{2}$B. in thickness you find the total Product to be 2302 Feet and 2 Inches.

Which 2302 must be divided by 272, the number of Feet contained in a Rod, as you read before.

Example.

126
272 ⟌ 2302 (8
272

Being divided, the Quotient is 8, and the remainder is 126, which is 8 Rods and 126 Feet and 2 Inches (if you add the 2 Inches that belonged to the 2302 Feet which you divided.)

Note, The whole Rod containing 272 Feet, the half Rod contains 136 Feet, and the Quarter of a Rod contains 68 Feet, for twice 136 is 272, and four times 68 is the same.

Then take 68 Feet or $\frac{1}{4}$ of a Rod out of the 126 Feet that remain, by substracting 68 From 126

68

And the remainer is 58 Feet.

So there is 8 Rods and $\frac{1}{4}$ and 58 Feet of $1\frac{1}{2}$B. work, in one Story of the design.

It is convenient before I proceed further, to add something more concerning measuring of Chimneys.

First Note, If you are to measure a Chimney standing alone or by it self, without any Party Wall being adjoyned,

Then

Then girt it about for the length, and the height of the Story is the breadth.

The thickneſs muſt be the ſame that your Jaums be, provided the Chimney be wrought upright from the Mantletree to the Ceiling, not deducting any thing for the vacancy between the Floor (or Hearth) and the Mantletree, becauſe of the gatherings of the breſt and wings to make room for the Hearth in the next Story.

Secondly Note, If the Chimney-back be a Party-wall, and the Wall being meaſured by it ſelf, then you muſt meaſure the depth of the 2 Jaums, and the length of the Breſt, which being added together, will be your length, and the height of the Story your breadth, at the ſame thickneſs your Jaums be.

Thirdly Note, When Chimneys are agreed for at a rate by the Fire-place or Chimney building, and the back of the Chimneys ſtand againſt a Wall (that you meaſure beſides) you muſt deduct 4 Inches in thickneſs out of the menſuration of the Wall againſt the Chimneys, which 4 Inches is allowed for the Backs of the Chimneys.

Fourthly Note, When you meaſure Shafts of Chimneys, girt them with a Line round about the leaſt place of them for the length, and their height ſhall be your breadth; And if they be 4 Inch work, then you muſt ſet down your thickneſs at 1B. work. But if they be wrought 9 Inches thick (as ſometimes they are when they ſtand high and alone above the Roof.) Then you muſt account your thickneſs 1½B. in conſideration of Withs and Pargetting and trouble in Scaffolding.

Obſerve

Observe these following Rules, which will much help you in casting up your Dimensions of any kind, and save you the labour of Division, to know how many Feet there is in any number of Inches that you multiply into Feet.

Let the first example be

	30	02
	18	03
	240	
	303	
	7	06
	550	06½

First, Having multiplied the whole Feet, In the next place you multiply the Inches into the Feet, saying 2 times 18 is 36 Inches; or thus, 18 Inches being a Foot and an half, say 2 Feet and 2 half Feet are 3 Feet, which being set down.

In the next place you come to multiply 3 into 30, which you need not multiply, for 3 being one fourth part of 12; say the ¼ of 30 Feet is 7 Feet and 6 Inches, which is a quicker way than to multiply 30 by 3, and divide the product by 12.

Let the second example be

	25	04
	42	05
	50	
	100	
	14	
	10	5
		1
	1074	6

The Feet being multiplied and set down, next you are to multiply 4 by 42; but 4 being ⅓ part of 12, say the one third of 4 is 1, and the one third of 12 is 4, which is (being added) 14 Feet.

In the next place you come to multiply 5 by 25, which you may say is 25 half Feet lacking 25 Inches, that is, 10 Feet and 5 Inches.

Or

Or thus, 24 Inches being 2 Feet, ſay 5 times 24 Inches is 10 Feet, then add the 5 Inches, it makes 10 Feet and 5 Inches. Laſtly, 5 by 4 is 20, which is 1 Inch and 8 parts, which 8 parts is not accounted, nor ſet down as you have read before.

Let the third example be

	100	7
	94	6
	400	
	900	
	50	
	47	
	7	10
		3
	9505	1

The Feet being multiplied you proceed to multiply 6 by 100, but 6 being the half of 12, ſay the half of 100 is 50 Feet, which being writ down, you proceed to multiply 7 into 94, then ſay the half of 94 is 47 Feet, which being ſet down, there is ſtill remaining 94 Inches, which you may ask your ſelf how many times 12 you can have in 94, your anſwer will be 7 times and 10 Inches remaining, therefore you muſt ſet down 7 Feet and 10 Inches. Laſtly, you proceed to multiply the Inches by the Inches, ſaying 7 times 6 is 42, which is 3 Inches and $\frac{1}{2}$; ſo the total Product is 9505 Feet and 1 Inch, as you ſee in the Margin above.

Let the fourth example be

	230	8
	310	9
	2300	
	690	
	266	8
	115	
	57	6
		6
	71679	8

The Feet being multiplied and ſet down, you proceed to multiply 310 by 8, but inſtead of multiplying, take the $\frac{2}{3}$ of 310 becauſe 8 is $\frac{2}{3}$ of 12; ſaying thus, the $\frac{2}{3}$ of 3 is 2, but the $\frac{2}{3}$ of 1 you cannot take in Integers, therefore you add a Cypher next to the Figure 2, and proceed to take the $\frac{2}{3}$ of 10, which is 6 Feet and 8 Inches; ſo you find the

$\frac{1}{3}$ of 310 Feet to be 206 Feet and 8 Inches, which being ſet down.

The next is 230 Feet to be multiplied by 9 Inches, which 9 Inches is $\frac{3}{4}$ of 12 Inches; therefore you muſt take $\frac{3}{4}$ of 230 Feet, ſaying, the half of 230 is 115, and $\frac{1}{2}$ of 115 is 57 Feet and 6 Inches, which being added to 115 makes 172 Feet and 6 Inches, as you ſee by caſting up the Dimenſion.

Let the fifth example be

	94	11
	48	10
	752	
	376	
	47	
	31	4
	44	9
	4635	1

The Feet being multiplied and ſet down, inſtead of multiplying 10 by 94, you muſt divide 10 into 2 parts (to wit) 6 and 4, 6 being $\frac{1}{2}$, and 4 being $\frac{1}{3}$ of 12; then ſay the $\frac{1}{2}$ of 94 is 47, and the $\frac{1}{3}$ of 94 is 31 Feet 4 Inches, which being ſet down, you proceed to know the Product of 48 Feet multiplied by 11 Inches, which to do ſay thus, it is 48 Feet lacking 48 Inches or 4 Feet, therefore I ſet down 44 Feet; behold the Example or Dimenſion caſt up in the Margin above.

Thus I have ſhewn you from 1 Inch to 12 Inches (beginning at 2 Inches, and ending at 11 Inches) how to work them into the Feet, to ſave Multiplication and Diviſion; which way, after you are expert in it, will ſave you ſome trouble and time in caſting up many Dimenſions.

Some

Some things to be obſerved in meaſuring of Brickwork.

Sometimes Walls are wrought 2 Inches thicker than any of theſe thickneſſes before ſpoken of.

Which 2 Inches ſerveth for a Water-table to the Wall, which is uſually ſet off, about 2 Feet above the ground; therefore you may meaſure the Wall at the the ſame thickneſs that it is above the Water-table, and add the 2 Inch work to it thus.

Suppoſe a Wall 20 Feet in length, and 2 B. thick above the Water-table.

After you have taken the Dimenſion of the Wall from the bottom to the height that you are to take it at 2 B. thick, then add 20 Feet in length by the height of the 2 Inch work (to wit) from the bottom to the ſetting off, or Water-table, which being halfed is ſo much 4 Inch work, and then reduce it to 1 ½ B. work.

Secondly Note, That all kinds of Ornamental work in Brick, is generally done at a Rate by the Foot, except there be an allowance of a ſumm of money over and above the Rate by the Rod, or except a good Rate is allowed by the Rod and all the Ornamental work to be meaſured into the Rod work.

By Ornamental work you are to underſtand ſtreight or Circular Arches over Windows or Doors, Facio's with or without Moldings, Architrave round the Windows or rubbed Returns, Friezes and Cornices, Ruſtick Quines *&c.* In brief, all kind of work that is hewed with an Axe, or rubbed upon a Stone,

is Ornamental work, and ought to be paid for beſides the Rod work.

Thirdly Note, when you meaſure Arches, either ſtreight or circular, you muſt meaſure them in the middle (that is to ſay) if a ſtreight Arch be 12 Inches in height or depth, you muſt meaſure the length of it in the middle of the 12 Inches, which length will be longer than if you meaſure it on the under ſide next the head of the Window, by ſo much as one ſide of the ſpringing of the Arch is skewed back from the up right of the Jaum.

Alſo in circular Arches, obſerve that the upper part of the Arch is more in length (being girt about) than the under part, becauſe it is a Segment of a greater circle cut off by the ſame right Line that the leſſer is, therefore it muſt be girt in the middle.

Fourthly Note, That Facio's, Architrave, Friezes, Cornices, Baſe Moldings and Plinths, are meaſured by the Foot, Line meaſure (or running meaſure as it is commonly called.)

Fifthly Note, You muſt take notice of the Jaums of Windows and Doors being ſplayed on the inſide of Buildings, at the rate by the Foot running meaſure.

Sixthly Note, Meaſuring of Tyling is the ſame as meaſuring of Roofing, which is taught at the beginning of this Tract in meaſuring of Carpenters work; only there is this difference, you muſt meaſure the length of all the Vallies, and add ſo many Feet as they are in length, to the number of Feet in the meaſurement of the Roof; you muſt alſo add the Eves and Barge-courſes, Tyling being done by the Square like Roofing.

You may alſo meaſure Tyling by the flat and half of the Building, after the ſame manner that you are

ſhewed to meaſure Roofing (in meaſuring of Carpenters work) adding the Barge-courſes and Eves, alſo there you may ſee how to meaſure Hipt Roofs.

Seventhly Note, That Brick-paving is done by the Yard, and is brought into Yards by dividing the Product of the Feet by 9, as you have read in meaſuring of Painting, Plaiſtering, *&c.*

Eighthly Note, The meaſuring of Gable-ends in Brickwork, is the ſame as in the meaſuring of Carpenters work, wherein you are taught to meaſure them.

Ninthly, Remember in the meaſuring of Walls joyning to each other Anglewiſe, that you take the length of one Wall to the outſide of the Angle, and the length of the other to the inſide of the Angle.

Tenthly, If you ſhould have a Gable end to meaſure, and you know the width of the houſe, or the length of the Baſe Line of the Gable end, and you deſire to know the length of the Perpendicular, you may find it by the Proportions of the Table that is in the meaſuring of Carpenters work, and the Rule of Three.

Or briefer thus, ſuppoſe the width of the Gable end to be 20 Feet, and you would know the length of the Perpendicular, take the length of the Rafter which is 15 Feet, being $\frac{3}{4}$ of the width (for if the Roof be true Pitch, $\frac{3}{4}$ of the breadth is the length of the Rafter) to which add half the length of the Rafter (*viz.*) 7 Feet and $\frac{1}{2}$, the Product is 22 Feet and $\frac{1}{2}$; the half whereof (to wit) 11 Feet and 3 Inches is the Perpendicular.

Although this is a way commonly uſed, yet it is a ſmall matter more than the length of the Perpendicular, as by the former Rule will appear.

Thus much will ſuffice a diligent and practical Reader concerning meaſuring, wherein to my knowledge I have not omitted any thing appertaining to the Science.

As to the meaſuring of Vaults and Arches, the following Treatiſes Entituled *The meaſuring of Plains* and *Bodies*, will inſtruct you.

And becauſe Brickwork is done at ſeveral rates *per* Rod, and many times there happens a number of Feet to remain, after you have divided your whole Product by 272, (to bring your number of Feet into Rods) it will be convenient to ſhew, How to find the Price of 1 Foot of Brickwork, or of any number of Feet, at any Rate *per* Rod.

Having the Price per *Rod given, and the number of Feet that you deſire to know what they come to, work thus.*

Firſt, Reduce the Price of a Rod into Shillings, by multiplying the Pounds by 20 (the number of Shillings in one Pound) and if the Price of a Rod be Pounds and Shillings, then add the Shillings to the Product of the Pounds, being multiplied by 20, ſo you will have the Price of a Rod in Shillings: Next multiply the number of Shillings by 12 (the Pence in one Shilling) and that brings the Price of a Rod into Pence. Then work according to the Rule of Three, ſaying.

Example.

Example. 1.

If I have 5 *l.* for a Rod of Brickwork, what must I have for 34 Feet, according to that Rate.

State the Question thus.

If a Rod which is 272 Feet give 5, what will 34 Feet give.

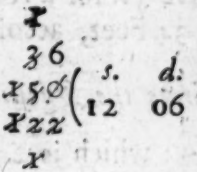
	l.	*s.*	
Multiply	5	00	
by		20	
The Product is		100	Shillings
Which multiply by		12	Pence
And the Product is		1200	Pence
Which multiply by		34	Feet
		4800	
		3600	
And the Product is		40800	

Which divide by 272

40800 (150
272
1360
1360

And the Quotient is 150 Pence for the 34 Feet, at 5 *l per* Rod.

Which 150 Pence, divide by 12, and the Quotient is 12 *s.* and $\frac{6}{12}$, or Six pence; the rate of the 34 Feet desired.

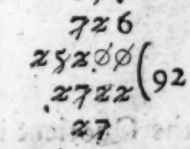


Example. 2.

If I have 5 *l.* 5 *s.* for a Rod, what comes 20 Feet to at that Rate.

State the Question thus, and it saves you the labour of multiplying by 20.

F.	S.	
If 272 give	105	what will 20 Feet give?
multiply by	12	Pence
	210	
	105	
The Product is	1260	the rate of a Rod in Pence, which
multiply by	20	the number of Feet, And
the Product is	25200	this Product divide by 272



The Quotient is 92 *d.* and $\frac{176}{272}$ of a Penny, which 92 *d.* being divided by 12 *d.* gives you 7 *s.* 8 *d.* and the

the Fraction remaining, namely $\frac{176}{272}$ is more than a half penny, and leſs than 3 Farthings.

So at 5 *l.* 5 *s.* *per* Rod, 20 Feet comes to 7 *s.* 8 *d.* 1 *ob.*

Example. 3.

If I have 5 *l.* 5 *s.* for one Rod of Brickwork, what muſt I have for 1 Foot.

	l.	
Multiply	05	
By	20	Shillings
The Product is	100	
Add the	5	Shillings
It is	105	Shillings
Which multiply by	12	Pence
	210	
	105	
The price of a Rod is	1260	Pence

Which ſhould be multiplied by the third number in the Queſtion (to wit) 1 Foot, but becauſe multiplying any number by 1 doth not encreaſe it.

Therefore divide 1260 *d.* by 272 Feet.

172 *l.*
*x*260 (4
272

And the Quotient is 4 *d.* $\frac{176}{272}$ of a Penny.

Then to know the value of the Fraction $\frac{176}{272}$.

Multiply

Multiply the Numerator 172 by 4 (the number of Farthings in one Penny) the Product is 688, which divide by the Denominator 272, and the Quotient is 2 Farthings, and the remainer $\frac{144}{272}$ of a Farthing, which Numerator 144 is more than half of the Denominator 272, and therefore more than half a Farthing.

Example.

The Numerator.	172	of the Fraction
Multiply by	4	Farthings
The Product is	688	which divide

by the Denominator 272.

And the Quotient is 2 Farthings and an half, and something more } 144 / 688 (2 Farthings $\frac{144}{272}$ / 272

Thus the rate of 1 Foot of Brickwork after the rate of 5 *l.* 5 *s. per* Rod, is found to be four pence half-penny half farthing; Which was required.

The same Rule holds in Tyling, or Carpenters work, only changing the number 272 (the Feet in a Rod) and instead of it use 100 (the number of Feet in a Square.)

The End of the Fourth Book.

MEASURING
OF
Superficial PLAINS,
BY
Vulgar ARITHMETICK
without Reducing the Integers into the leaſt Denomination.

TOGETHER
With Directions for the Meaſuring
OF
LAND.

The Fifth BOOK.

By *VEN. MANDEY.*

LONDON,
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MEASURING OF Superficial PLAINS.

CHAP. I.

By measuring of Plains is meant the finding the Area or Content of Superficial Figures, such as are Quadrates, Triangles, Circles, &c. and as a Line is made by the motion of a Point, so likewise a Superficie is made by the motion of a Line.

IN measuring of Plains; the first Figure to begin with (as being most easie) is a Quadrate or Square, which is a Figure enclosed with four equal Sides, making four right Angles.

The next is an Oblong or long Square, which is a Figure having four right Angles, and four Sides, but not equal or of one length. Two of which Sides being opposite are of one length and Parallel; and the other

other two ſides being of another Length are oppoſite and Parallel.

The meaſuring of theſe two kinds of Plains, you have been already ſhewn in the preceding Tract of Meaſuring, (to wit, by multiplying the length by the breadth.)

PROP. I. *Fig. I.*

To meaſure (or find the Content of) a Rhombus, being a Figure like a Quarry of Glaſs, having four equal Sides, and two Pair of equal Angles.

SUppoſe the Rhombus ABCD in *Fig.* I. one ſide whereof is 5 Feet and 4 Inches, take this for your Length.

Then for the Breadth take the neareſt diſtance between any two of the ſides, as ſuppoſe the Prick Line E F being 5 Feet, multiply theſe two, and the Product is 26 Feet 8 Inches, which is the Area or Content of the Rhombus ABCD.

Otherwiſe thus.

Draw the Diagonal Line AC, which divides the Rhombus into two Triangles, next draw the Perpendicular Line DB.

Then take the Diagonal Line AC for your Length, and the half Length of the Perpendicular for your Breadth, and multiply them, and the Product is the Content.

See the Examples cast up both ways.

	Feet	In.
The length of one side AB (the other three being each the same length) is	5	4
The nearest distance (or EF) is	5	0
	25	
	1	8
Being cast up the first way, the Content is	26	8

The second way.

	F.	In.	P.
The length of the Diagonal Line AC is	9	2	0
The half length of the Perpendicular is	2	10	11
	18	4	
	7	6	
		1	8
		8	3
			1
The same Content as before	26	8	0

The multiplying of the parts of Inches into the Feet and Inches; you are taught in the foregoing Tract of Measuring of Glaziers Work.

PROP.

PROP. II. *Fig. II.*

To measure (or find the Content of) a Trapezium or four sided Figure, two sides whereof are Parallel.

SUppose the Trapezium ABCD in *Fig.* II. whose longest side AB is 9 Feet 4 Inches $\frac{1}{2}$; the side parallel to it is DC, being 7 Feet 2 Inches and $\frac{10}{12}$ of an Inch; the breadth being taken the neareſt way, is the Prick Line whose length is 6 Feet and $\frac{1}{4}$ of an Inch.

Add the two parallel sides together, whereof take half the Summ for your length; then take the neareſt distance between those two sides that are neareſt together for your breadth, and multiply them, the Product whereof is the Content.

Example.

	F.	I.	P.	
The longeſt side AB, is	9	4	6	
The shorteſt parallel side DC, is	7	2	10	
Those two being added, is	16	7	4	
The mean length being one half, is	8	3	8	
This being mult. by the neareſt distance	6	0	3	
	48			Sec.
	1	6		
		4		
		2		9
				2
The Content required is	50	9	0	11

The

The 11, beyond the place of parts of Inches is $\frac{11}{12}$ parts of $\frac{1}{12}$ part of an Inch or $\frac{11}{144}$ parts of an Inch, which is not considerable, except the Superficies that you measure be some rich or costly thing.

Note, That I account or suppose the Inch to be divided into 12 parts, for two reasons. The first is, the more parts it is divided into, the nearer to the truth you may take the length and breadth of any Superficies or Solid, and likewise proceed the nearer to the true Content.

The second reason is, because the parts of Inches thus divided, have the same proportion to the Inches, that the Inches have to the Feet, and by that means these parts may be multiplied into the Feet and Inches, which according as the Inch is divided on our common Rulers into 8 parts; those parts cannot be multiplied into the Feet and Inches, but the length, and likewise the breadth of any Superficies must be reduced into the least denomination, and then multiplied each by other, which causeth a great deal of multiplication, and also of division, before you arrive at the Superficial Area or Content, as you may see in the preceding Tract at the beginning of Measuring of Glaziers Work, from Pag. 216, to Pag. 219, where I have proved this way of multiplying the parts of Inches into the Feet and Inches to be true, both by Decimal Arithmetick, and also by reducing the length and breadth into the least denomination (to wit, Quarters of Inches.)

And indeed as Mr. *Oughtred* saith in his Circles of Proportion, the dividing of the Inches on the Rulers into halfs and quarters, and half quarters, is most inartificial; and I suppose was done before Decimal Arithmetick was brought to light. Which if the Foot were

were divided into 10 Parts (which may as well be called Inches, as now it is divided into 12 Parts) and each of those parts into 10 parts, and the length and breadth of Plains or Solids taken therewith and set down Decimally, and cast up by Decimal Arithmetick: The Science of Measuring would be much easier than now it is, and save a great deal of labour.

Note, That a four sided Superficies which hath none of the sides parallel; likewise every plain Figure of more sides than four being proposed to be measured, must with Diagonal Lines be divided into Triangles.

And Note, That every such Figure containeth so many Triangles as it hath sides, abating two out of the number (to wit, if the Figure to be divided into Triangles have six sides, it will contain four Triangles; if seven sides, five Triangles); then these Triangles are to be measured severally, and their Contents being added, gives the Content of the Irregular Figure.

Prop. III. *Fig. III.*

To measure any Triangle.

Multiply the longest side (being usually called the Base) by half the Perpendicular (which is a Line let fall Perpendicular from the Angle opposite to the Base) and the Product is the Content of the Triangle.

Suppose a Triangle A B C in *Fig. III.* whose Base A C is 7 Feet, the side C B 6 Feet, and the side B A 5 Feet, and the Perpendicular which is the Prick-

line 4 Feet 2 Inches, the half whereof is 2 Feet 1 Inch, which being multiplied by the Base Line 7 Feet, makes 14 Feet 7 Inches for the Content of that Triangle.

Example.

	F.	I.
The length of the Base is	7	0
Half the length of the Perpendicular is	2	1
The Content of the Triangle is	14	7

Otherwise thus.

Multiply the whole Perpendicular by half the Base.

Example.

	F.	I.
The whole Perpendicular is	4	2
Being multiplied by half the Base	3	6
	12	6
	2	1
The Content (as before) is	14	7

PROP. IV. *Fig. IV.*

The side of an Equilateral Triangle being given, to find the Perpendicular Arithmetically.

LEt the side of the Triangle given be A C in *Fig. IV.* whose length is 8 Feet.

First, Square the side, then Square half the Base, and

and ſubtract that from the Square of the ſide, and the ſquare Root of the Remainer is the length of the Perpendicular.

Example.

	F.	I.
The given ſide is A C	8	0
Which to ſquare, multiply by	8	0
The Square of the ſide A C is	64	0
The whole Baſe is 8, the half whereof is	4	0
Which to ſquare, multiply by	4	0
The Square of half the Baſe is	16	0
Which ſubſtract from	64	0
The Square of the given ſide	16	
and the Remainer is	48	0

The ſquare Root whereof is 6 Feet 11 Inches and $\frac{3}{12}$ parts of an Inch *Fere*, being the length of the Perpendicular required.

It is ſuppoſed that the Reader hereof can extract ſquare and cube Roots, if he cannot, he may learn it in Mr. *Wingates* Arithmetick, in which Book they are plainly and briefly Demonſtrated; or he may work by Logarithms.

PROP. V. *Fig. IV.*

The Perpendicular and Side of an Equilateral Triangle being given, to find the Content of that Triangle.

IN the former Diagram, *Fig. IV.* the Perpendicular 6 Feet 11 Inches $\frac{1}{12}$ *Fere* and the Side 8 Feet being given, the Content is required.

Multiply the whole of either by half the other, and the Product is the Content required.

Example.

	F.	I.	Pa.	
The whole Base or side is	8	0	0	
The half of the Perpendicular is	3	5	7	*Fere*
	24			
	3	4		
		4	8	
The Content of the Triangle is	27	8	8	*Fere*

If you multiply the whole Perpendicular 6 Feet 11 Inches, by half the side 4 Feet, it produceth the same as before (to wit) 27 Feet 8 Inches $\frac{1}{12}$ of an Inch almost, for *Fere* signifies almost or near.

PROP.

PROP. VI. *Fig. V.*

The Perpendicular and Baſe of a right Angled Triangle being given, the Content of that Triangle is required.

IN *Fig. V.* the Perpendicular F G, being 12 Feet, and the Baſe C E 25 Feet, is given, and the Content required.

Multiply the whole of either by half the other, as 12 Feet, by 12 Feet 6 Inches, or 25 Feet by 6, the Product is 150 Feet, the Superficial Content required.

Or multiply the whole by the whole, the Product is 300, whereof take half, it is 150 as before.

Or thus, without the Perpendicular.

Multiply the two ſides, 15 by 20, the Product is 300, whereof take half for your demand; or multiply half of one ſide by the whole of the other ſide, the Product is 150 as before.

Theſe Rules hold good in thoſe right Angled Triangles, whoſe longeſt ſide (or Baſe) being ſquared, the Product is as much as the Square of both the other ſides, as in *Fig. V.* the ſquare of C E is 625, the ſquare of C F is 225, and the ſquare of F E is 400, which two laſt Squares being added, make 625 the ſquare of the Baſe; ſo likewiſe in a right Angled Triangle, whoſe ſides are 6 Feet, 8 Feet, and 10 Feet, it holds the ſame proportion, for the ſquare of 10, which is 100, is as much as the ſquare of 6, which is 36 and the ſquare of 8 which is 64 being added together.

PROP. VII. *Fig. VI.*

The Perpendicular and Base of an Isoceles Triangle given, to find the Area or Superficial Content.

IN *Fig. VI.* add all the three sides together, whereof take half, then take the difference of each side from that half, and multiply those three differences one by another, the Product whereof multiply by the half summ of the three sides, and the square Root of that Product is the Superficial Content.

Example.

The 3 sides being added make 56, the half whereof is 28, then the difference of the side A B (being 20) from 28 is 8; the like difference is between the side B C and 28, namely 8, and the difference between the side A C (being 16) and 28 is 12. Now these 3 differences being multiplied one by another, namely 8 by 8 is 64, and 64 by 12 makes 768, which being multiplied by 28 the half summ of the 3 sides, the Product is 21504, whereof the square Root is 146 Feet 7 Inches 8 Parts and $\frac{2}{12}$ of a Part, or 146 Feet 7 Inches and $\frac{72}{144}$ of an Inch.

Which Rule is general for all right lined Triangles whatsoever.

PROP.

PROP. VIII. *Fig VII.*

To find the Perpendicular of any Triangle Arithmetically, the ſides being given.

IN *Fig. VII.* let the ſides given be E D 6 Feet, D F 8 Feet, and F E 10 Feet, it is required to find the Perpendicular.

Square the 3 ſides ſeverally, then add the Square of the Baſe (or longeſt ſide) to the Square of one of the other two ſides, as to the Square of E D, from whence ſubſtract the Square of the remaining ſide, and from the remainer take half, which divide by the Baſe, the Quotient thereof being ſquared, deduct or ſubſtract it from the Square of the ſhorteſt ſide, and the ſquare Root of the remainer is the length of the Perpendicular.

Example.

The ſquare of E D is 36, the ſquare of D F is 64, and the ſquare of F E is 100; then the ſquare of the Baſe 100 being added to the ſquare of E D 36, makes 136, from whence the ſquare of D F being ſubſtracted which is 64, the remainer is 72, whereof the half is 36, which being divided by the Baſe 10 produceth 3 Feet $\frac{3}{5}$; or 3 Feet 7 Inches 2 parts and $\frac{2}{5}$ of a part, for the leſſer Segment of the Baſe E G; the Square of which Segment is 12 Feet $\frac{18}{25}$, or 12 Feet 11 Inches and 6 parts, and being Subſtracted from the Square of E D 36, the remainer is 23 Feet $\frac{7}{25}$ or 23 Feet and $\frac{7}{12}$ of an Inch +, Whoſe Square Root 4 Feet 9 Inches

and 7 parts +, is the length of the Perpendicular D G required.

PROP. IX. *Fig. VIII.*

The side of a Pentagon being given, to find the Area or Content.

IN *Fig. VIII.* let the given side be 10, it is required to find the Area of that Pentagon.

Describe an Isoceles Triangle on any side of the Pentagon whose top is the Centre of the Pentagon, as A D B, whose Perpendicular being found is 6 Feet 10 Inches and 5 parts + of an Inch divided into 12 parts, which being multiplyed in halfe the Perimetry 25, Produces 171 Feet 8 Inches and 5 parts, for the Area required.

This Rule is general in all kind of regular Polygons of how many sides soever, as well for their Superficial Content, as finding their Perpendicular.

But becaufe it is troubleſom finding the Centre Geometrically I will here ſhew how to find it by Arithmetick.

The side of a Pentagon being given to find the Radius of a Circle inscribed within that Pentagon.

The Rule is this, as 182 to 125 ſo is the ſide of the Pentagon to the Semidiameter of a Circle inſcribed in it.

Example.

The ſide given is 10 F. what is the neareſt diſtance from the Centre of the Pentagon to one of the ſides.

Multiply

Multiply and Divide according to the Rule of Three and you will find 6 Feet 10 Inches and $\frac{5}{7}$ Parts. +

Prop. X. *Fig.* IX.

The Diameter of a Circle being given, to find the Circumference thereof Arithmetically.

THe proportion of the Diameter to the Circumference was first found out by *Archimedes*, who brought the length of the Perimeter of a Circle within the limits of Numbers very little differing from the truth, Demonstrating the same to be less than three Diameters and a seventh part, but greater than three Diameters and $\frac{10}{71}$ parts of the Diameter, so that supposing the Radius to consist of 10000000 equal parts, the Arch of a Quadrant will be between 15714285 and 15704225 of the same parts. But since *Ludovicus Van Cullen* hath come yet nearer to the truth and pronounced from true Principles that the Arch of a Quadrant (putting as before 10000000 for the Radius) differs not one whole Unity from the number 15707963. But for our purpose we shall make choice either of the proportion in whole Numbers by *Archimedes*, which is as 7 to 22, or of the proportion which *Metius* hath given, which is as 113 to 355, so is the Diameter to the Periphery or Circumference.

Let the Diameter given be 14 Feet, in *Fig. IX.* It is required to find the Circumference thereof.

Multiply the Diameter given (being 14) by 22, the Product is 308, which divided by 7, bringeth 44 Feet for the Circumference required; or multiply the Diameter by $3\frac{1}{7}$ the Product is 44 as before; but this makes

makes the Circumference a ſmall matter to great, as you may ſee if you multiply 14 by 355, the Product is 4970, which being divided by 113, the Quotient is 43 and $\frac{111}{113}$ the remainer, which is 111 parts of an Unit (whether it be a Foot, or an Inch or any other Meaſure) divided into 113 parts.

If the Circumference be given and the Diameter required, it appeareth by this Rule that the Circumference 44 being multiplyed by 7 and the Product divided by 22, brings 14 the Diameter.

PROP. XI. *Fig. IX.*

The Diameter and Circumference of a Circle being given to find the Area or Superficial content thereof Arithmetically diverſe ways.

LEt the Diameter of the Circle be 14 as in *Fig. IX.* and the Circumference thereof 44. It is required to find the Superficial Content.

Multiply half the Diameter by half the Circumference it ſhews the Content of any Circle.

Thus the half Circumference is 22, which being multiplied by the Semidiameter 7, the Product is 154, the Superficial Content required.

Or multiply the whole circumference 44 by the Semidiameter 7, the Product will be 308, whereof take half, which is 154 as before.

Or multiply the Square of the Diameter 196 by 11, the Product will be 2156, which divide by 14, brings 154 as before.

PROP.

PROP. XII. *Fig. IX.*

Having only the Diameter of a Circle given, to find the Content.

SAy as 7 is to 22, so is the Square of the Semidiameter to the Content of the Circle.

Let the Diameter given be 14 as in *Fig. IX.*

Then say by the Rule of Three, if 7 give 22, what will 49 give (which 49 is the Square of the Semidiameter); multiply the third Number 49 by the second Number 22, the Product is 1078, which divide by the first Number 7, and you have 154 for the Content, as before.

PROP. XIII. *Fig IX.*

Having the Circumference given, to find the Content.

THe Rule is as 88 (which is 4 times 22) is to 7, so is the Square of the Circumference to the Content.

Example.

Let the Circumference be 44, the Square thereof is 1936; then say by the Rule of Three.

If 88 give 7, what will 1936 give.

Multiply 1936 by 7

The Product is 13552

Which

Which divide by the first Number 88, and the Quotient is 154 Feet.

43
573
23552
8888 (154 Feet, the Content of the Circle.
88

PROP. XIV. *Fig. IX.*

The Content of a Circle being given, to find the Diameter.

THe Rule is as 22 to (4 times 7 which is) 28, so is the Content to the Square of the Diameter.

Example.

Let the Content given be 154, and it is required, to find the Diameter.

If 22 give 28, what will 154 give? multiply and divide, and you will find 196, the Square Root whereof is 14, the Diameter required.

PROP. XV. *Fig. IX.*

The Content of a Circle being given, to find the Circumference.

THe Rule is, as 7 is to (4 times 22 which is) 88, so is the Content to the Square of the Circumference.

Example.

Example.

Let the Content given be 154, and the Circumference required to be found.

Then say by the Rule of Three.

If 7 give 88, what will 154 give? multiply and divide, and you will find 1936, the Root whereof being 44, is the Circumference required.

Prop. XVI. *Fig. IX.*

The Content of a Circle being given; To find the Side of a Square, the Content of which Square shall be equal to the Content of the Circle.

Extract the Square Root of the Content given, and that Root is the Side of the Square required.

Example.

Let the Content given be 154, the Square Root whereof is 12 Feet 4 Inches $\frac{11}{12}$ parts of an Inch, which is the Side of a Square, whose Content is equal to the Content of the Circle given.

Prop.

Prop. XVII.

The Diameter of any Circle being given, to find the Side of a Square, the Area of which Square ſhall be equal to the Area of the Circle of the given Diameter.

THis Propoſition may be ſolved ſeveral ways, as firſt according to *Archimedes* his Proportion in whole Numbers thus.

As 7 is to 22, ſo is the Square of the Semidiameter, to the Content required.

Let the Diameter given be 7 Feet, the Semidiameter will be 3 Feet 6 Inches, the Square whereof is 12 Feet 3 Inches; which being multiplied by 22 Feet, the Product is 269 Feet and 6 Inches, which divided by 7, makes 38 Feet 6 Inches for the Content of the Circle, which Content is a ſmall matter more than the true Content.

Secondly, According to *Metius* his Proportion thus.

As 113 to 355, ſo is the Square of the Semidiameter to the Content required.

Example.

The Square of the Semidiameter aforeſaid, is 12 Feet 3 Inches, which being multiplied by 355 Feet, produces 4348 Feet 9 Inches, which Product being divided by

by 113 Feet, makes 38 Feet 5 Inches 9 parts and $\frac{2}{113}$ of a part, which Content is leſs than the Content produced from *Archimedes* Proportion, by more than $\frac{2}{144}$ parts of a Superficial Foot, and the greater the Diameter is, the more the difference will be.

Or thus; As 452 (which is 4 times 113) is to 355; ſo is the Square of the Diameter, to the Content required.

Thirdly, It may be ſolved by Decimal Arithmetick thus, as 1 Foot, to 3.1416 Feet, ſo is the Square of the Semidiameter to the Content required.

Example.

3.1416 Fe. being multiplied by 12.25 Fe. produceth 38.484600 Feet, which ſhould be divided by the firſt number 1, but becauſe 1 doth neither increaſe any number being multiplied by it nor diminiſh any Number being divided by it, therefore we find the Content to be 38.484600 Feet, which being reduced to Feet Inches and Parts, is 38 Feet 5 Inches 9 Parts 9 Seconds and 4 Thirds.

Fourthly, It may be ſolved Decimally thus.

As 114.5915492 Feet, is to 360 Feet, ſo is the Square of the Semidiameter to the Content required.

Example.

360 Feet being multiplyed by 12.25 Feet, produceth 4410.00 Feet which being divided by 114.5915492 Feet produceth 38.48451 Feet, which being reduced, is 38 Feet 5 Inches 9 Parts 9 Seconds and 2 Thirds, which is a ſmall matter leſs than the former Content.

But

But peradventure the Reader hereof may not underſtand Decimal Arithmetick, I will therefore Exhibit one way more by Vulgar Arithmetick, for the Solving of this Propoſition; which is this.

Multiply the Diameter given, by 10 Feet 7 Inches 7 Parts 5 Seconds; and divide the Product by 12 Feet, It produces the Side of a Square, which being multiplyed by it ſelf, is the Content of the Circle required.

Which I look upon to be as quick a way, as by Decimals, eſpecially if the Diameter given, conſiſts of Feet, Inches, Halfs, Quarters, *&c.* and the Solution be required in the ſame; And cometh within a Trifle as near the truth.

And here I ſhall a little inſiſt upon the multiplying the Fractional parts of Inches into one another, For two Reaſons.

Firſt, as being a way very uſeful, and faves a great deal of Labour (to wit) of reducing the Integers into the leaſt Denomination (or Fractional Parts) it being the uſual way ſo to do, among thoſe who underſtand not Decimal Arithmetick.

Secondly, Becauſe hitherto I have had occaſion only to multiply twelfths of Inches into Feet and Inches; and having occaſion now to multiply other twelfths into thoſe twelfths, and other twelfths into theſe twelfths, *&c.* which I name thus.

Feet, Inches, Parts, Seconds, Thirds, Fourths, &c.

And here Note, that as a Foot contains 12 Inches Line meaſure, ſo an Inch is ſuppoſed (or accompted) to contain 12 Parts Line meaſure (although upon the Rulers, the Inches are divided but into 8 parts, or half quarters) and each part to contain 12 Seconds, and each Second to contain 12 Thirds, and each Third to contain

contain 12 Fourths, *&c.* And so the Proportion might be continued if there were occasion, to Fifths, Sixths, *&c.*

But according to Superficial measure, a Foot contains 144 Inches, and an Inch 144 parts, and a Part 144 Seconds, *&c.*

Also observe, that each place of twelfths is distinguished by having a Letter put over it; thus over the Feet write F, over Inches I, over Parts P, over Seconds S, over Thirds T, over Fourths Fo. *&c.* as beneath.

And for the better understanding how to multiply the Fractions or twelfths into one another, observe the Rules in the Following Table.

	F.	I.	P.	S.	T.	F
1. *Note*, That In. mult. into F. every 12 of the Prod. is and any Number less than 12, is Inches.	1					
2. In. mult. into (or by) In. every 12 of the Prod. is and any Number of the Prod. less than 12, are Parts.	0	1				
3. Parts mult. into Feet, every 12 of the Product is and any Number under 12, are Parts,	0	1				
4. Pa. multiplyed into In. every 12 of the Prod. is and any Number of the Prod. under 12, are Seconds	0	0	1			
5. Pa. multiplyed by Pa. every 12 of the Prod. is and any number less than 12, are Thirds.	0	0	0	1		
6. Sec. multiplyed into Fe. every 12 of the Prod. is and any Number less than 12, are Seconds.	0	0	1			
7. Sec. mult. into In. every 12 of the Product is and any Number less than 12, are Thirds.	0	0	0	1		
8. Sec. multip. into Pa. every 12 of the Product is and any Number less than 12, are Fourths.	0	0	0	0	1	
9. Se. multiplyed into Se. every 12 of the Prod. is and any Number less than 12, are Fifths.	0	0	0	0	0	1
10. Th. mult. into Fe. every 12 of the Prod. is and any Number less than 12, are Thirds	0	0	0	1		
11. Th. mult. into Inches every 12 of the Prod. is and any Number less than 12, are Fourths.	0	0	0	0	1	
12. Th. mult. into Pa. every 12 of the product is and any Number less than 12, are Fifths.	0	0	0	0	0	1
13. Fo. multip. into Fe. every 12 of the product is and any Number less than 12, are Fourths.	0	0	0	0	1	

Farther in Fractional Parts you need not proceed.

Here *Note*, if the greateſt Denomination of the Dimenſion to be caſt up, be Inches; and the Content be required in Superficial Inches; Then, Parts multiplyed into Inches, every 12 of the Product is one Inch; and Seconds multiplied into Parts, every 12 is 1 Part, *&c.*

Explanation of the Table in 3 or 4 Examples.

Firſt, Suppoſe you have 6 parts to be multiplyed into 7 Feet.

Say, 6 times 7 is 42, then ask your ſelf how many times 12 is in 42, and the anſwer is 3 times, and 6 remaining; therefore according to the 3 Rule of the preceding Table, ſet 3 in the place of Inches, Noted with the Letter I, and the 6 that remains, being leſs than 12, you muſt Write in the place of Parts, Noted above the Dimenſion with the Letter P.

Secondly, Suppoſe 8 Thirds to be multiplied into 8 Inches, ſay 8 times 8 is 64, which contains 5 twelves, and 4 remaining, therefore according to the 11 Rule, ſet 5 in the place of Thirds Noted by T, and the 4 that remains ſet in the place or Column of Fourths, Noted with Fo.

Thirdly, Suppoſe you have 10 Seconds to be multiplied by 11 Parts, ſay 10 times 11 is 110, which contains nine times 12 and 2 remaining; then according to the 8 Rule of the foregoing Table; ſet down 9 in the place of Thirds, Noted on the top with T, and the 2 that remains ſet in the place of Fourths, Noted by Fo, *&c.*

Theſe

Theſe being premiſed, I come now to ſolve the foregoing *Prop.* (to wit) the Diameter of a Circle being given whoſe length is 7 Feet. It is required to find a Square whoſe Content ſhall be equal to that Circle.

The Proportion is, as 12 Feet, is to 10 Feet 7 Inches 7 Parts 5 Seconds; ſo is the Diameter to the Side of a Square, or the Square of the Diameter to the Content required.

The Rule thus; multiply the Diameter by 10 Feet 7 Inches 7 Parts 5 Seconds, and divide the Product by 12 Feet. The Quotient together with the Remains if there happen to be any, is the Side of a Square, which being multiplyed in it ſelf, is equal to the Content required.

Example.

	F.	I.	P.	S.	T.	Fo.
The length of the Diam. given is	7	0	0	0	0	0
Which multiply by	10	7	7	5		
The Prod. of 10 Fe. by 7 Fe. is	70					
The Prod. of 7 Inches by 7 Fe. is	4	1				
The Prod. of 7 Parts by 7 Feet is	0	4	1			
The Prod. of 5 Sec. by 7 Feet is	0	0	2	11		
The Total Product is	74	5	3	11		

Which Product according to the foregoing Rule, muſt be divided by 12. Therefore divide the 74 Feet by 12 Feet, thus,

12(2 Inches.
74(6 Feet.
12

And the Quotient is 6 Feet and 2 remains, which is 2 Inches, ſo the Side of the Square equal to the Circle, is 6 Feet 2 Inches 5 Parts 3 Seconds 11 Thirds; For inſtead of dividing the 5 Inches 3 Parts 11 Seconds (that belonged to the 74 Feet in the Total Product) by 12, only do thus, remove them one place forwarder towards the Right Hand, it is the ſame as if you had divided them by 12; ſo then the 5 Inches will ſtand in the place of Parts, and be 5 Parts, and the 3 Parts will be 3 Seconds, and the 11 Seconds will be 11 Thirds; Which obſerve as a general Rule in any Number to be divided after this manner.

When you have divided the Feet by 12, if there happen to be no Inches remaining, then ſet down your Feet under the place of Feet, and put a Cypher in the place of Inches, and then ſet your other Fractions next; as in this laſt Example, if there had been no remainer beſides the Quotient, then you muſt have put 0 in the place of Inches, and then the 5 Inches which belonged to the 74 Feet, will be changed into 5 Parts, as if they had been divided by 12, And likewiſe the 3 Parts will be turned into 3 Seconds, &c.

F.I.

	F.	I.	P.	S.	T.	F.
So that 74 Feet 5 Inches 3 Parts 11 Seconds, being divided by 12, is	6	2	5	3	11	
Which is the Side of a Square; which multiply by	6	2	5	3	11	
The Product of 6 Feet by 6 Feet, is	36					
The Product of 2 Inches by 6 Feet, is	1					
The same again is	1					
The Product of 2 Inches by 2 Inches, is	0	4				
The Product of 5 Parts by 6 Feet, is		26				
The same again is		26				
The Product of 5 Parts by 2 Inches, is	0	0	10			
The same again is			10			
The Product of 5 Parts by 5 Parts, is				21		
The Product of 3 Seconds by 6 Feet, is		1	6			
The same again is		1	6			
The Product of 3 Seconds by 2 Inches, is				6		
The same again is				6		
The Product of 3 Seconds by 5 Parts, is				1	3	
The same again is				1	3	
The Product of 11 Thirds by 6 Feet, is			5	6		
The same again is			5	6		
The Product of 11 Thirds by 2 Inches, is				1	10	
The same again is				1	10	
Farther you need not proceed, the value of the Products being inconsiderate.						
The Total Product is	38	5	9	10	7	2

Which is the Content required, and agrees with the Decimal way within $\frac{1}{17\frac{1}{2}\frac{1}{8}}$ parts of an Inch.

Although I have set the Product of each multiplication one under another for Demonstration; yet never-

nevertheless when you are perfect in casting up, you may set the Products of 4 or 5 multiplications in one level line.

Let a second Example be.

A Diameter given, whose length is 42 Feet, and the Content of that Circle is required.

According to *Archimedes* his Proportion in whole Numbers, working as is shewn before, the Content will be 1386 Feet.

And according to *Metius*, in whole Numbers the Content will be found to be 1385 Feet and $\frac{10}{113}$ of a Foot, which reduced, is 1385 Feet 5 Inches 3 Parts 8 Seconds.

Let us try what the Content will be our way.

	F.	I.	P.	S.	T.	F.
The Diameter is	42	0	0	0	0	0
Which multiply by	10	7	7	5		
The Product of 10 Feet by 42 Feet, is	420					
The Product of 7 Inches by 42 Feet, is	24	6				
The Product of 7 Parts by 42 Feet, is	0	24	6			
The Product of 5 Seconds by 42 Feet, is			17	6		
The Total Product is	446	7	11	6		

Which 446 Feet, Divide by 12.

x
*x*8(2 Inches.
446 (37 Feet.
xxx
x

The

The Quotient is 37 Feet and 2 Inches, to which add the 7 Parts 11 Seconds 6 Thirds, by changing their places one Degree forwarder toward the Right Hand, (instead of dividing them by 12) and it will be

	F.	I.	P.	S.	T.	Fo.
For the Side of the Square	37	2	7	11	6	0
Which to Square, multiply by	37	2	7	11	6	0
The Prod. of 7 Feet by 37 Feet is	259					
The Prod. of 30 Feet by 37 Fe. is	111					
The Prod. of 2 Inch. by 37 Feet is	6	2				
The same again is	6	2				
The Prod. of 2 Inches by 2 Inch. is			4			
The Prod. of 7 Parts by 37 Feet is		21	7			
The same again is		21	7			
The Prod. of 7 Parts by 2 Inches is			1	2		
The same again is			1	2		
The Prod. of 7 Parts by 7 Parts is				4	1	
The Product of 11 Sec. by 37 F. is			33	11		
The same again is			33	11		
The Prod. of 11 Sec. by 2 Inches is				1	10	
The same again is				1	10	
The Product of 11 Sc. by 7 Parts is					6	5
The same again is					6	5
The Prod. of 11 Sec. by 11 Sec. is						10
The Prod. of 6 Thirds by 37 Fe. is				18	6	
The same again is				18	6	
The Prod. of 6 Thir. by 2 Inches is					1	
The same again is					1	
The Prod. of 6 Thirds by 7 Parts is						3
The same again is						3
The Content or Total Product is	1385	5	8	0	1	2

Which Content, is four Parts more than the Content according to *Metius*'s proportion, but near four parts less than the Content according to *Archimedes*'s Proportion.

And by the way Note, that a Circle of 14 Feet Diameter, contains 4 times as much as a Circle that is 7 Fe. Diameter. The Rule is thus, the Diameter 7 is contained in the Diameter 14, two times, now the Square of 2 is 4, therefore a Circle whose Diameter is 14, contains a Circle whose Diameter is 7, four times.

Likewise the Diameter in the last Example being 42, contains the Diameter 7, six times, for the Square of 6 is 36, therefore a Circle whose Diameter is 42, contains 36 Circles of 7 Feet Diameter a Piece.

For if you multiply 38 Feet 5 Inches 9 Parts 10 Seconds + (being the Content of a Circle whose Diameter is 7) by 36, you will find it produce the Content of a Circle whose Diameter is 42.

Example.

	F.	I.	P.	S.	T.
A Circle whose Dia. is 7 Fe. the Con. is	38	5	9	10	6
Which multiply by	36				
	228	0	0	0	0
	114				
	15	27	30	18	0
A Circ. whose Di. is 42 F. contains	1385	5	7	6	0

Let the Third Example be.

A Diameter whose length is 38 Feet 6 Inches and an half and half a quarter, and it is required to find the Content of that Circle. The

The half Inch is 6 Parts, but the half quarter is $\frac{1}{8}$ Part of an Inch, which must be reduced to T[illegible]fths thus: Multiply the Numerator of the Fraction (to be reduced) by 12, and divide the Product by the Denominator of the same, And the Quotient is the Numerator of a Fraction that hath 12 for his Denominator, and if any Number remain besides the Quotient, multiply that remainer by 12, and divide the Product by 8, so long until you have no remainer besides the Quotient.

Example.

$\frac{1}{8}$ of an Inch is to be reduced; Multiply the Numerator 1 by your new Denominator 12, it is still 12, divide it by the Denominator 8, the Quotient is 1, and 4 remains, the 1 in the Quotient is 1 Part, which add to the $\frac{1}{2}$ Inch or 6 Parts, and then it makes 7 Parts; Then multiply the remainer 4 by 12, the Product is 48, which divide by the Denominator 8, and the Quotient is 6 and no remainer, which is 6 Seconds. So $\frac{1}{8}$ of an Inch being reduced to Twelfths is 1 Par. 6 Sec.

	F.	I.	P.	S.	T.
Now I set down the Diameter thus	38	6	7	6	0
Which I multiply by	10	7	7	5	

380

5 | 3 | 6

22 | 2 | 4 | 1

5 | 10

22 | 2 | 4

3 6

5

15 | 10

3 6

2 6

The Product is	409	11	11	3	0

Which

Which 409 divide by 12

x4(1 In.
4ø9(34 F.
xzz
z

	F.	I.	P.	S.	T.
The Side of the Sq. is	34	1	11	11	3
Which multiply by	34	1	11	11	3
	136	31	2		
	1022	10	1		
	2	10		11	10
		31	2	11	10
				10	
		31	2	11	
		31	2	11	10
				8	6
				8	6
The Content required is	1167	3	11	8	6 10

This last *Example* could not be Solved so near the truth by Vulgar *Arithmetick* any other way, without reduceing the 38 Feet 6 Inches and half Inch, into half quarters of Inches, which is a long way about.

PROP. XVIII.

The Superficial Content of a Circle being given to find the Diameter.

LEt the Content given (being found according to *Archimedes* Proportion) be 346 Feet 6 Inches; and the Diameter required.

The proportion is, as 11 is to 14, so is the Content given, to the Square of the Diameter required.

The

The Rule thus, Multiply the Content given, by 14, the Product whereof divide by 11, and the Square root of the Quotient is the length of the Diameter.

Example.

Multiply the Content given, being 346 Feet 6 Inch. by 14, the Product is 4851, which being divided by 11, the Quotient is 441 for the Square of the Diameter, the Root whereof is 21, the length of the Diameter required.

Or thus, and more exact, as 355 to 452, so is the Content given, to the Square of the Diameter.

Example.

Multiply the Content given, being 346 Feet 6 Inch. by 452, the Product is 156618, which being divided by 355, produces 441 Feet and $\frac{61}{355}$ of a Foot, which when reduced, is 441 Feet 2 Inches 1 Part 6 Seconds, the Square Root whereof, is 21 Feet 1 Inch 5 Parts 6 Seconds, the Diameter required.

To those who understand Decimals, the Proportion for Solving the 18 Prop. is this;

As 1.00000, is to 1.27324, so is the Content given, to the Square of the Diameter,

Therefore if you multiply 1.27324, by the given Content, the Square Root of that Product will be the Diameter required.

PROP.

Prop. XIX.

The Diameter of a Circle being given; To find the Side of a Square which may be Inſcribed within that Circle.

THe Rule is this, Square the Diameter, Then take one half of the Product, Then find the Root of that Product; It will be the Side of the Square deſired;

Example.

Let the Diameter given be 28 Feet 3 Inches, The Square whereof is 798 Feet 9 Parts; The half whereof is 399 Feet 4 Parts and 6 Seconds; The Square Root whereof is 19 Feet 11 Inches 8 Parts 5 Seconds +, For the length of the Side of the Square required.

Decimally thus, as 1.000000 is to .707106, So is the Diameter to the Side required.

Therefore if you multiply the ſaid .707106 by the Diameter given, the Product will be the Side of the inſcribed Square required; for as you read before, If you ſhould divide the Product by 1.000000 it will ſtill be the ſame.

Prop. XX. *Fig. X.*

The Diameter and Arch Line of a Semicircle being given; To find the Area thereof.

LEt A B C in *Fig. X.* be a Semicircle given, whoſe Diameter is A C, and the Arch Line A B C, It is required to find the Area of that Semicircle.

Multiply

Multiply half the Arch Line 11 by the Semidiameter 7, The Product will be 77 for the Area required.

PROP. XXI. *Fig. X.*

The Semidiameter and Arch Line of a Sector of a Circle being given, To find the Area.

IN *Fig. X.* let B C D be the Sector of a Circle whose Semidiameter is D C, or D B, and the Arch Line B C, and it is required to find the Area.

Multiply the Semidiameter 7, by half the Arch Line B C 5 Feet 6 Inches, the Product is 38 Feet 6 Inches for the Area required.

PROP. XXII. *Fig. X.*

Any Segment or part of a Circle being given to find the Content.

IN *Fig. X.* let A F B E be the Segment of a Circle, The Content whereof is required.

First find out the Center of the Arch Line A F B (by the 21 Prop. of the 20 Page of Geometry) then draw the Lines D A and D B, and cast up the whole Figure A F B D, as in the last Prop, which will be 38 Feet 6 Inches, then find the Superficial Content of the Triangle A B D, which is 24 Feet 5 Inches 9 Parts, and deduct the same out of the whole Content 38 Feet 6 Inches; the Remains is 14 Feet and 3 Parts, for the Content of the given Segment, which was required.

By

By this Rule (obſerved with Diſcretion) may all manner of Segments or parts of a Circle, whether greater or leſſer than a Semicircle be eaſily meaſured.

PROP. XXIII. *Fig. X.*

To find the Diameter of a Circle, by having one part of the Diameter given, alſo having the length of the Chord croſſing the Diameter in the given part.

IN *Fig X.* let F E be the part of the Diameter given, alſo let A B be the given Chord which cuts off the given part of the Diameter; it is required to find the whole Diameter Arithmetically.

F E the part of the Diameter given is 2 Feet and $\frac{7}{12}$ of an Inch, A B the Chord interſecting it, is 9 Feet 10 Inches $\frac{10}{12}$ of an Inch.

The Rule is thus. Square one half of the Chord, and divide the Product by the part of the Diameter given, which Quotient being added to the ſaid part, is the length of the Diameter required.

Example.

The whole Chord A B is 9 Feet 10 Inches 10 Parts, the half whereof is 4 Feet 11 Inches 5 Parts, the Square of which is 24 Feet 6 Inches 2 Parts and 4 Thir. which being divided by 2 Feet 7 Parts (the part of the Diameter given) the Quotient is 11 Feet 11 Inches 8 Parts, to which 2 Feet 7 Parts being added, makes 14 Feet for the length of the Diameter required.

PROP.

PROP. XXIV. *Fig. X.*

Any Segment of a Circle being given, whose Chord Line doth not exceed the Chord of the Quadrant of the same Circle; to find the Content without finding the Diameter, and without describing any more of the Circumference.

ALthough by the Precedent Prop. XXII, any Segment of a Circle may be measured; yet because the finding of the Center, and measuring of the Sector, and then measuring the Triangle and Substracting it from the Sector, requires a great deal of Trouble and time; I will here shew how to do it with less trouble, and very near the truth in small Segments.

Let the Segment given be the same (whose Content was found by 22 Prop.) in *Fig. X.* whose Chord Line A B is 9 Feet 10 Inches and 9 Parts; and the part of the Diameter F E (cut off by the Chord Line) is 2 Feet 7 Parts and 6 Seconds, the Content of which Segment A F B E is required.

Note, if the Versed Sine, *viz.* the Line F E be not given, you must divide the Chord A B in the middle, and from that middle point as at E, raise a Perpendicular to the Arch Line.

Then take the whole length of the Chord A B, and two Thirds of the length of the Line F E, to which two Thirds add 7 Parts, then multiply those two lengths, and the Product gives you the Content.

Example

Example.

The Verſed Sine F E is 2 Feet 7 Parts 6 Seconds; two third parts whereof is 1 Foot 4 Inches 5 Parts, to which 7 Parts being added, makes it 1 Foot 5 Inches, which multiplied by the Chord A B, (namely) 9 Feet 10 Inches 9 Parts, produces 14 Feet and 2 Parts for the Content: which is 1 Part leſs than the Content found by the 22 Prop.

Another Example.

In *Fig.* XI. the Semidiameter is 14 Feet, And the Content of the Quadrant A B C D will be found by the former Rules according to *Metius* his Proportion, to be 153 Feet 11 Inches 1 Part 6 Seconds.

Now it is required to find the Content of the Segment A B C E.

By ſubſtracting the Content of the Triangle A C D which is 98 Feet 4 Parts and 5 Seconds, from the Content of the Quadrant which is as above, there remains 55 Feet 10 Inches 9 Parts, For the Content of the Segment A B C E.

Let us ſee what the other way will produce.

The length of part of the Diameter B E is 4 Feet 1 Inch 2 Parts, Two third parts whereof is 2 Feet 8 Inches 9 Parts 4 Seconds, to which add 7 Parts, and it makes 2 Feet 9 Inches 4 Parts 4 Seconds, which being multiplyed by the Chord Line A C, whoſe length is 19 Feet 9 Inches 7 Parts, the Product is 54 Feet 11 Inches 6 Parts for the Content of the Segment.

A

A third Example.

Let the Content of the two little Segments A B, B C, in *Fig. XI*, be required.

The Line F G is 1 Foot, two third parts whereof is 8 Inches, to which, 7 Parts being added, makes it 8 Inches 7 Parts, which being multiplyed by 10 Feet 10 Inches, the length of the Chord Line A B, produces 7 Feet 8 Inches 11 Parts, for the Content of the Segment A F B G, the other Segment B C being equal to this, add 7 Feet 8 Inches 11 Parts to the Content of the firſt, and the Product is 15 Feet 5 Inches 10 Parts, for the Content of both the little Segments.

The Triangle A B C E whoſe Content is 40 Feet 6 Inches 5 Parts, being Deducted from the Content of the Segment A B C E, which is 55 Feet 10 Inches 9 Parts, there remains 15 Feet 4 Inches 4 Parts, for the Content of the two little Segments; which is half an Inch leſs than was found before.

PROP. XXV. *Fig. XII.*

An Irregular Plat or Figure being given, to find the Area or Content thereof.

LEt a b c d e f g h i in *Fig. XII*, be an irregular Plat whoſe Superficial Content is required.

Reduce the ſame into as many Trapezias as it will contain, as firſt the Trapezium a b c d, Secondly a d e f, Thirdly a f g i, and there remains the Triangle i g h. In which three Trapezias, draw the Diagonals b d, d f, and f i, which ſhall be as Common Baſes to each Tri-

angle on either side; on which Bases let fall the Perpendiculars from the several Angles at a, c, e, and g, then in every Trapezium, take the length of the Base by it self, and the length of the two Perpendiculars thereon falling joined together, in one Number by it self, then multiply the half of the one, in the whole of the other, and the Product is the Area of that Trapezium, which reserve by it self; and working in like sort with the rest, and Lastly, the Triangle i g h, Collect all their Products together, which shall shew the Content required.

Prop. XXVI.

To find the Superficial Content of any Oval.

TAke both the Diameters and multiply them one into another, then extract the Square Root of that Product, and that Root will be the Diameter of a Circle, whose Content will be equal to the Content of the Oval. For the Area of a Circle, is to the Area of an Ellipsis, as the Square of the Diameter of that Circle, is to the Rectangled Figure of the Transverse and Conjugate Diameters of the Ellipsis, by the 6th of *Archimedes* Conoids and Spheroides.

Example.

Let *Fig. XIII.* be an Oval whose longest Diameter is 10 Feet, and the shortest Diameter is 6 Feet, and the Content required.

Multi-

Multiply the Length 10 by the Breadth 6, the Product is 60, the Square Root whereof is 7 Feet 8 Inches 11 Parts 5 Seconds, which is the length of the Diameter, the Content of whose Circle is equal to the Content of the Oval; then multiply the Diameter by 10 Feet 7 Inches 7 Parts 5 Seconds (as you were shewn at the 5th *Example* of Prop. XVII. of this) and the Product is 82 Feet 4 Inches 6 Parts 3 Seconds, which divide by 12, it produces 6 Feet 10 Inches 4 Parts 6 Seconds 3 Thirds, which being multiplyed by it self (*viz.* Squared) the Product is 47 Feet 1 Inch 5 Parts 11 Seconds, the Content of the Oval required.

See the whole Work.	F.	I.	P.	S.	T.
The Length of the Oval	10	0	0	0	0
The Breadth of the Oval	6	0	0	0	0
Being multiplyed, the Product is	60	0	0	0	0
The Square Root whereof is	7	8	11	5	
Which being the M. Diam. mult. by	10	7	7	5	
	70				
	6	8	8	6	5
	4	1	6	5	
		9	2	3	4
		4	1	2	11
			4	8	2
			4	2	4
			2	11	
The Product is	82	4	6	3	2

Which Product, divide by 12.

(1 Inch.
2(0
8 2 (6 Feet
4 2

		F.	I.	P.	S.	T.
And it is		6	10	4	6	3
Which multiply by		6	10	4	6	3
		36	8	4	1	4
		5	2	3	4	2
		5	2	3	4	2
				3	5	3
				3	5	2
					1	6
					1	6
						2
The Content of the Oval is		47	1	5	11	3

In Decimals, thus: Multiply the length of the Oval by the breadth, and divide the Product by 1.27324 Foot; it gives the Content.

PROP. XXVII. *Fig XIV.*

To find the Superficial Content of a Cylinder, the Diameter being given.

Note, That a Cylinder is a solid Body, which may well be represented by a Roll, either of Timber or Stone, such as are used in Gardens for the rolling of Walks.

The Proportion is, *As* 7 *to* 22, or *As* 113 *to* 355; *So* is the Diameter and length of the Side multiplied one by the other, *to* the superficial Content of the outside of the Cylinder, besides the two Bases or ends.

The

The Rule, this: Multiply the Diameter by the Length or Side; then multiply that Product by 355, the Product whereof divide by 113, and it gives you the Content of the round Superficies.

Let *Fig. XIV.* be a Cylinder, whose Diameter is 4 feet, and the Side or Length 10 feet, and the Content required.

First, Multiply the Diameter 4 by the Length 10, the Product is 40; which multiply by 355, and the Product is 14200; which divide by 113, and the Quotient is 125 feet, and 75 remaining, which is $\frac{75}{113}$ of a foot; and being reduced, is 125 F. 7 I. 11 P. 6 S. for the Content required.

And if you had wrought after the Proportion of 7 to 22, the Content would have been 125 F. 81. 7 P. *ferè*, which is a small matter more than the former.

If you have the circumference and length of a Cylinder given, and the Content required,

Multiply the Circumference by the Length, and the Product is the Superficial Content.

Example.

Let the Circumference and Length of the Cylinder, *Fig. XIV.* be given, and the Content required.

Multiply the Circumference 12 F. 6 I. 9 P. 7 S. by the Length 10 F. and the Product is 125 F. 7 I. 11 P. for the Content required.

If you are minded to add the Superficial Content of the two Bases or Ends of the Cylinder, you will find the Content of them as you are shewn before in measuring of Circles.

PROP. XXVIII. *Fig. XV.*

To find the Superficial Content of a Cone.

A Cone is a Body which hath a Circle for its Base, from whence it diminisheth equally (like a round Spire of a Steeple) 'till it end in a Point.

Let *Fig. XV.* be a Cone, the Circumference of whose Base is 24 F. and the length of the Side 10 F. and the Content required.

If you require the Content of the whole Cone, that is to say, the Content of the Base, as well as the Content of the outside; then multiply the whole Side by half the Compass or Periphery of the Base, and to the Product, add the Content of the Plain of the Base.

Example.

The whole Circumference of the Base being 24 F. the half thereof is 12 F. which being multiplied by the Length 10 F. produces 120 F. for the Content of the outside, besides the Base: Then by the 13*th* Prop. of this, find the Content of the Circle whose Circumference is 24, and add it to the former Content, and the Product is the Superficial Content of the Cone, together with its Base.

Example.

	F.	I.	P.	S.
The Cont. of the Cone without the Base, is	120	0	0	0
The Content of the Base, is	45	9	9	9
The Content of the Cone, together with its Base, is	165	9	9	9

PROP.

Prop. XXIX. *Fig. XVI.*

To find the Superficial Content of a Pyramid.

As a Cone hath a round Baſe, and diminiſheth equally 'till it end in a Point; ſo a Pyramid hath an angular Baſe of 3, 4, 5, 6, or any number of Sides, and diminiſheth to a Point at the top.

Let *Fig. XVI.* be a Pyramid, whoſe Baſe is a Square whoſe Side is 5 feet, and the length from the Baſe to the top, 10 feet, and the ſuperficial Content required.

Add the four ſides of the Baſe together, of which Product take half; then multiply that half, by the ſide or length.

Example.

One ſide being 5 feet, the four ſides added, make 20 feet, the half whereof is 10 feet; which multiplied by 10 feet the length, produces 100 feet for the ſuperficial Content, beſides the Baſe: If you are minded to add the Content of the Baſe to it, which is 5 times 5 feet, or 25 feet, then the ſuperficial Content of the Pyramid, together with the Baſe, will be 125 feet.

Likewiſe if the Baſe be Triangular, you muſt add the three ſides of the Baſe together, and take half of the Product, which multiplied by the length, gives you the Content: Then if you would add the Content of the Baſe to the Content of the outſide, meaſure it as you are ſhewn before, in meaſuring of Triangles.

Alſo if the Baſe be a Pentagon, take half the Product of the five ſides of the Baſe (being added together)

and multiply it by the length, and the Product gives you the Content: If you are minded to add the Base, find the Content, as you are shewn at *Prop. IX.* of this.

PROP. XXX.

To find the Superficial Content of a Globe or Sphere.

MUltiply the Diameter or Axis, by the Circumference, and the Product will be the superficial Content of the round Body or Globe.

Example.

Let the Diameter be 14 feet, the Circumference will be 44 *ferè*, which being multiplied one by the other, the Product is 616 feet for the Content of the Superficies of the Globe.

Or find the Content of the Circle that hath the same Diameter as the Globe, and multiply that Content by 4, and the Product is the Superficial Content required.

Example.

A Circle whose Diameter is 14, the Content is 154, which multiplied by 4, the Product is 616, as before.

PROP.

Prop. XXXI. *Fig. XVII.*

To find the Superficial Content of a Fragment, or part of a Globe.

THE Superficie of any Fragment, or Portion of a Globe, is to the Superficie of the remaining Portion; as the Portion of the Axis or Diameter cut off, is to the remaining Portion of the ſame.

Example.

Let there be in *Fig. XVII.* a Portion of a Globe given A B C, whoſe part (or Segment) of the Axis (or Diameter) is 5 feet, and the whole Diameter 14 feet, and the ſuperficial Content of the circular Part is required.

Say by the Rule of Three, If 14 give 616, what will 5 give? Multiply 616 by 5, the Product is 3080; which divide by 14, and you have 220 feet for the ſuperficial Content of the Segment, or part of the Globe.

I ſhall now conclude the meaſuring of *Plains*, with ſome directions whereby to meaſure *Land*.

Wherein note, That 5 ½ yards, or 16 ½ feet in length, is called (according to Statute) a *Pole* or *Perch*; and one Perch in breadth, and 40 in length, is called a *Rood*; and 4 Perches in breadth, and 40 in length, is one *Acre*: So that an Acre contains 160 ſquare Perches, half an Acre 80 Perches, and a quarter of an Acre, commonly called a Rood, 40 Perches.

In

In measuring of Land, it is usual to take the Lengths and Breadths with a Chain, the length of which Chain some make to contain 4 Poles (or Perches) as Mr. *Gunter* and others, and some make it to contain 2 Poles, as Mr. *Rathborn* and others.

The Chain which I would advise is thus:

To contain 2 Perches (or 33 Feet) in length, each of which Perches, divide into 12 equal parts, or (as I may call them) Links, and each Link into 12 equal parts, with little Wyars or Notches crosswise.

Thus the length of a Perch will be divided into 144 parts, and the whole Chain, consisting of 2 Perches, will contain 288 parts; in the middle of which Chain it will be convenient to have a pretty large Ring, that so you may distinguish where one Perch ends, and the other begins: Also midway between that large Ring, and one end of the Chain, hang a Curtain-Ring; and likewise hang another Curtain-Ring midway between the other end and the large Ring: So the whole Chain will be divided into four equal parts or half Perches, by the three Rings.

The Chain being thus divided, you have every whole Pole equal to 12 Links, or 144 Parts; every $\frac{3}{4}$ of a Pole equal to 9 Links, or 108 Parts; every half Pole equal to 6 Links, or 72 Parts: And lastly, every quarter of a Pole equal to 3 Links, or 36 Parts.

The Chain being divided after this manner, you have only three Denominations to set down, namely, *Chains*, *Links*, *Parts*, if you will measure so near the truth; but in most Mensurations you need proceed no farther than to Chains and Links, and then you have but two Denominations to set down.

Also the manner of casting up the Dimensions, is the same as is shewn all along before, only altering the

the two first Denominations, to wit, whereas you did set over the Dimensions, F, I, P, *&c.* signifying Feet, Inches, Parts, so in measuring of Land, you must set over the Dimensions, C, L, P, signifying Chains, Links, Parts: For as a Foot contains 12 Inches, and an Inch 12 Parts, Line measure (or in length); so likewise a Pole or Chain contains 12 Links, and one Link 12 Parts.

You must remember to double the number of your Chains when you set them down, because a Chain contains 2 Poles, or Perches.

Suppose in *Fig. V.* C F E B to be a piece of Land lying in form of a long Square, and the length F E, being measured with the Chain, to contain 8 Chains, 6 Links, and 6 Parts; because the double of 8 is 16, I set it down thus . . .

Also the breadth E B set thus, containing

	C.	L.	P.
	16	6	6
	14	9	3
	64	4	6
	167	7	1	6
Which I multiply as if they were Feet, Inches, and Parts	12	4	4	6
				1
And the Product in square Perches, is	244	4	0	1

Which 244 Chains or Perches you must divide by 160 to bring them into Acres, as you read above, that an Acre contains 160 square Poles or Perches.

So you have 1 the Quotient, which is 1 Acre, and the 84 that remains being Perches, divide by 40, the number of Perches in a Rood, and you have the Quotient 2 Roods and 4 Perches remaining.

Thus the Content of that piece of Land is found

(84
244 (1
160

(4
84 (2
40

found to be 1 Acre, 2 Roods, 4 Perches, and the 4 Links which remains in the Multiplication, is one third part of a Perch.

Again.

Suppose *Fig. VII.* to be a Field, enclosed with three sides, like a Triangle E D F, and you require to know the Content thereof.

Let the Side E D be 120 Chains or Perches, the Side D F 160 Chains or Perches, and the Side F E 200 Perches, and the Perpendicular D G 95 Perches.

(As you were shewn in measuring of Triangles) take half the Base or Side E F, which is 100 Perches, and multiply it by the Perpendicular D G 95 Perches.

Example.

	C.	L.	P.
The length of half the Base E F, is	100	0	0
Which multiplied by the Perpendicular	95	0	0
	500		
	900		
The Product is, in square Perches . .	9500	0	0

Which being divided by 160, produces 59 Acres, one Rood and an half; (for if you divide 60 by 40, it is one and a half) which is the Content of that Triangular piece of Land.

(6
x4Ø(0
Ø4ØØ(59
x6ØØ
x6

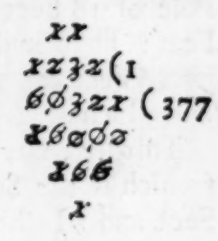
Example.

Example.

A ſquare piece of Land being in breadth 120 Chains, 9 Links, 6 Parts, and in length 125 Chains, 6 Links, and 2 Parts, and the Content is required.

	C.	*L.*	*P.*
The double of 120 is 240, ſet thus	240	9	6
The double of 125 is 250, ſet it thus	250	6	2
	12000	4	6
	4⁰0	125	1
	120		3
	187	6	
		40	
The Product in Perches is . .	60321	7	10

Which divide by 160, and the Quotient is 377 Acres; and 1 remaining which is 1 Perch, to which add the 7 Links that are in the Product of the Multiplication, and the whole Content will be 377 Acres, 1 Perch, and 7 Links, which is half a Perch, and 1 Link.



The 10 Parts being but of a ſmall value, is ſeldom ſet down.

For as much as in ſome Countries they allow 18 *Feet to the Pole or Perch, and in other Countries more, as* 20 *or* 24; *and all allow* 160 *of their Perches to an Acre: it will be convenient here to ſhew how to reduce one kind of Meaſure into another.*

Example.

Example.

Suppose it were required to reduce 4 Acres, 3 Roods, and 4 Perches of Statue Measure (that is, 16 Feet and an half to the Pole) into customary Measure of 18 Feet to the Pole.

First, Reduce your given Quantity, 4 Acres, 3 Roods, and 4 Perches, into the least denomination, to wit, Perches; by multiplying the 4 Acres by 160, (the number of Perches in one Acre) and the Product is 640 Perches in the 4 Acres: Then multiply the 3 Roods by 40 (the number of Perches in one Rood) and the Product is 120 Perches in the 3 Roods. Lastly, add the 640 Perches, and the 120 Perches, also the 4 Perches together, and the total Product is 764 Perches, being contained in the given number, *viz.* 4 Acres, 3 Roods, and 4 Perches.

Secondly, Square the two Poles or Perches, that is to say, multiply each in it self: As first, multiply the Pole of 18 Feet by 18 Feet, and the Product is 324 Feet: Then multiply the Pole of 16 Feet and an half by 16 Feet and an half, and the Product is 272 Feet and 3 Inches.

Thirdly, Say by the Rule of Three: *As* 324 (which is the Square of the 18 feet Pole) *Is* to 272 Feet and 3 Inches (which is the Square of the 16 Feet and an half Pole): *So* is 764 Perches (the number given) *To* a fourth Number required.

First, multiply 764 by 272 Feet and 3 Inches, and the Product is 207999, which divide by the first number in the Question, namely, 324, and the Quotient is 641 Perches, $\frac{215}{324}$ of a Perch, which being abreviated is $\frac{25}{36}$ of a Perch.

Fourthly

Fourthly, To reduce the Perches into Acres, divide them by 160; ſo 641 Perches being divided by 160, you find 4 Acres and 1 Perch remaining.

Thus you ſee, that 4 Acres, 3 Roods, and 4 Perches of Statute Meaſure, of 16 Feet and an half to the Pole, being reduced, is 4 Acres, 1 Perch, and $\frac{11}{36}$ of a Perch, cuſtomary Meaſure of 18 Feet to the Pole.

An Example of the whole Work.

The quantity given is 4 Acres, 3 Roods, 4 Perches of Statute Meaſure, to be reduced to cuſtomary Meaſure of 18 Feet to the Pole or Perch.

Firſt, reduce the Quantity given into the leaſt denomination, namely, Perches.

Firſt, Reduce the . .	4 Acres.	
The Perches in 1 Acre is . .	160;	Being multiplied,
The Perches in 4 Acres is . .	640	
Secondly, Reduce the . .	3 Roods.	
The Perches in 1 Rood is . .	40;	Being multiplied,
The Perches in 3 Roods is . .	120	

The third number 4 being the leaſt denomination, namely, Perches, needs no reducing.

Now add the Perches together.	640
	120
	4
And you find the Quantity given contains	764 Perches.

The

The Quantity given being brought into Perches, the enſuing Work will be, to find the number of Superficial Feet that each Pole contains.

And, Firſt, The Pole of	18 Feet,
Being ſquared, or multiplied by . . .	18
	144
	18
Contains, or the Product is	324 Feet.

	F.	I.
Secondly, The Pole of	16	6
Being ſquared, or multiplied by . .	16	6
	96	
	168	
	8	3
Contains, or the Product is . . .	272	3

Thirdly, By the Rule of Three, ſay, *As* 324 Feet, *is to* 272 Feet, 3 Inches: *So* is 764 Perches, *to* a fourth Number required.

	F.	I.
Multiply the ſecond Number . .	272	3
By the third	764	
	1088	
	1632	
	1904	
	191	
The Product is	207999	

Which

Which divide by the first number 324, and the Quotient is 641 Perches, and $\frac{215}{324}$ parts of a Perch, which abreviated is $\frac{35}{36}$ parts of a Perch, which is almost a Perch.

Fourthly, Having now reduced the Quantity of given Perches (namely, 764 of Statute Measure) to 641 Perches, and $\frac{35}{36}$ of a Perch, of customary Measure of 18 Feet to the Pole or Perch; the next work is to bring these Perches into Acres.

```
(3
 6(1
 1353(5
 207999 (641
 32444
  322
   3
```

Which to do, divide them by 160, and the Quotient (as you see in the Margin) is 4 Acres and 1 Perch, to which add the $\frac{35}{36}$ of a Perch, and then you have 4 Acres, 1 Perch, and $\frac{35}{36}$ of a Perch, of customary Measure (of 18 Feet to the Pole) contained in 4 Acres, 3 Roods, and 4 Perches of Statute Measure of 16 Feet and an half to the Pole.

```
(1 Perch.
641 (4 Acres.
160
```

A second Example.

Suppose on the contrary, that 4 Acres, 1 Perch, and $\frac{35}{36}$ of a Perch, of customary Measure of 18 Feet to the Pole, were required to be reduced into Statute Measure, of 16 Feet and an half to the Pole.

As in the last Example, First reduce the given Quantity into the least Denomination, to wit, Feet; for in this Example, the least Denomination is Parts of a Perch, which are Feet.

Therefore first multiply the 4 Acres, by 160, and the Product is 640, to which add the 1 Perch given, and it is 641 Perches.

Secondly, To bring these Perches into Feet, you must multiply them by the number of Feet contained in a Perch, which, as you found before in the last Example, by multiplying 18 by 18, the Product was 324: So then you must multiply 641 Perches by 324 Feet, and the Product will be 207684 Feet: To which you must add the Fraction $\frac{35}{36}$, whose value you must find thus:

First, multiply the Numerator 35 by the number of Feet in a Perch, to wit, 324, and the Product is 11340, which divide by the Denominator 36, and the Quotient is 315 Feet, which is the value of the Fraction $\frac{35}{36}$, and must be added to the 207684 Feet, and then the whole number of Feet will be 207999. Thus you have the Quantity given reduced into the least denomination, namely, Feet.

And because the Statute Pole 16 Feet and an half is a Fraction, and being squared, it produces a Fraction, and must be the first number in the Question, according to the Rule of Three, and by consequence the Divisor of the Division; which cannot be in Vulgar Arithmetic to divide by an Integer mixt with a Fraction: We must therefore reduce the Feet into Inches, by multiplying 207999 by 12, the Product is 2495988 Inches; likewise multiply 272 Feet by 12, and the Product is 3264, to which add the 3 Inches that belong to the 2-2 Feet, and it is 3267 Inches.

Then divide 2495988 by 3267, the number of Inches in a Statute Perch, and the Quotient is 764 Perches of Statute Measure, which divide by 160, and you have 4 Acres in the Quotient, and 124 Perches remaining; which 124 divide by 40, the Perches in a Rood, and you have 3 Roods and 4 Perches.

So

So that 4 Acres, 1 Perch, and $\frac{1}{16}$ of a Perch, of Cuſtomary Meaſure being reduced to Statute Meaſure, contains 4 Acres, 3 Roods, and 4 Perches, which was required to be done.

After the ſame method may any Cuſtomary Quantity whatſoever be reduced.

I ſhall not need to mention any more Examples of Land that is circular, or irregular Plats, having already ſhewn how to find their Contents.

Note: When you meaſure the length or breadth of a Field with your Chain, you muſt meaſure in a ſtreight Line as near as you can, otherwiſe you will make the Length longer than it is.

I ſhall conclude the meaſuring of Superficial Plains with this advice:

If you intend to learn any thing that is herein contained, you muſt not only Read, but likewiſe put Pen to Paper, and caſt up the Examples, and alſo other Examples of your own propoſing, and with a little Uſe mixt with Diligence, you will ſoon attain to your deſire: And you will find this way of multiplying Fractions into Integers very pleaſant and brief, in reſpect of the other way in Vulgar Arithmetick, whereby the Integers muſt be reduced into the leaſt Denomination, which is 4 times more labour than this way.

The End of the Fifth Book.

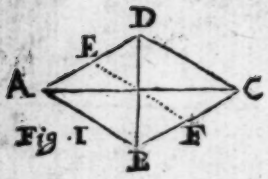


Fig. I

Fig II

Fig. III.

Fig. IV.

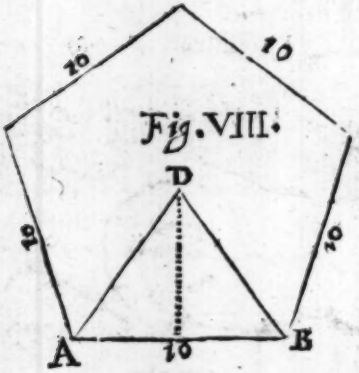


Fig. VIII.

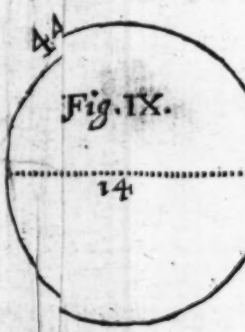


Fig. IX.

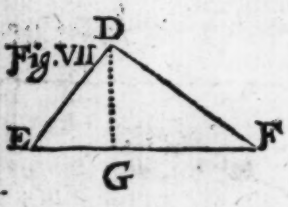


Fig. VII

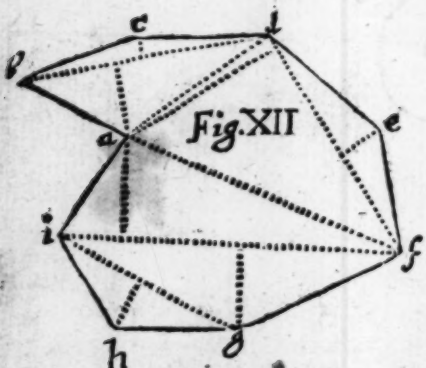


Fig. XII

Fig. XIII

Fig. XIV

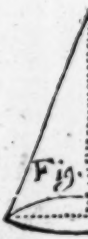
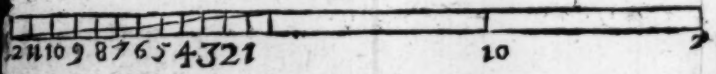


Fig.

The Scale appertaining to theese Figures



Place this between P

g. IV.

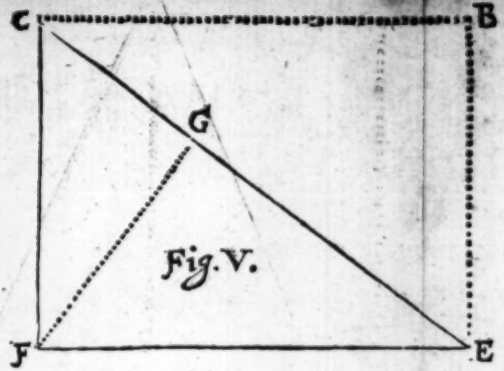


Fig. V.

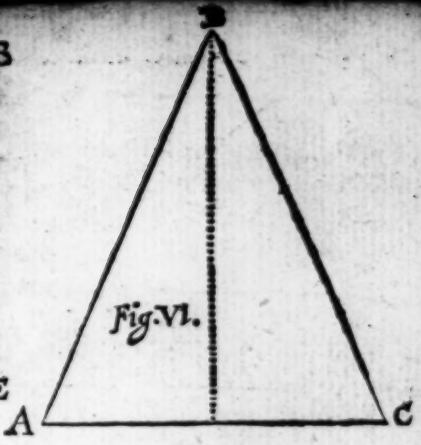


Fig. VI.

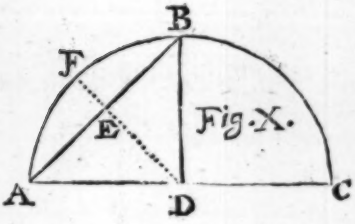


Fig. X.

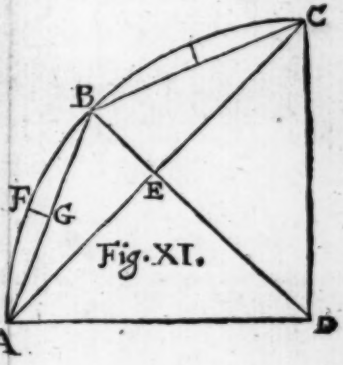


Fig. XI.

Fig. XV

en Page 322 and 323.

MEASURING OF SOLIDS.

The Sixth BOOK.

By VEN. MANDEY.

LONDON,
Printed *Anno Dom.* M DC. LXXXI.

MEASURING OF SOLIDS.

BY measuring of Solids is meant the measuring of Stone, Timber, digging of Earth, and all solid Magnitude whatsoever.

And as from the motion of a Line is made a Superficies, so likewise from the motion of a Superficies is made a Solid or Mathematical Body.

And as a Superficies consists of Length and Breadth, so a Solid or Mathematical Body consists of Length, Breadth, and Depth (or Thickness.)

There is no farther progress; for which way soever a Solid Body is moved, a Solid Magnitude will be thereby described.

In measuring of Solids, I shall begin with the Cube, it being most easie to measure.

A

A Cube is a Rectangular or Square Solid, that hath an equal Length, Breadth, and Depth, and is comprehended under ſix equal Squares, and may well be repreſented by a Dye, which is it ſelf a little Cube.

And as in meaſuring of Plains you only multiply the Breadth by the Length, for the gaining of the Content.

So in meaſuring of Solids, you muſt firſt multiply the Breadth by the Length, for the gaining the ſuperficial Content; and then multiply that Superficial Content by the depth or thickneſs of the Solid Body, and the Product is the Solid Content.

Of Timber or Stone being Square and equal Sided.

PROP. I. *Fig. I.*

Fig. I. *being a Cube* 12 *Inches in Length, and* 12 *Inches in Breadth, and* 12 *Inches in Depth, what is the Solid Content thereof?*

FIrſt multiply 12 Inches the Length by 12 Inches the Breadth, and the Product is 144 Inches for the Superficial Content; which multiplied by 12 Inches the Depth, produces 1728 Inches for the Solid Content of that Cube.

Example.

	Inches.
The Length	12
The Breadth	12
	24
	12
The Superficial Content is	144
Which multiply by the Depth . . .	12
	288
	144
And the Solid Content required, is .	1728

Where, by the way, you may take notice, That as a Superficial Foot contains 144 Superficial Inches, so likewise a Cubical or Solid Foot contains 1728 Cubical Inches, being produced as above.

So that by a Foot of Timber or Stone, or any other Solid Material in whatsoever kind of Solid it be found, is understood a Cube containing 1728 Cubical Inches, and consequently half a Foot Solid contains 864 Cubick Inches, and a quarter of a Foot contains 432 Cubick Inches.

Prop. II.

If the Side of a Cube of Timber or Stone be 4 Feet 5 Inches, what is the Content of that Cube?

First find the Content of the Side, by multiplying 4 Feet 5 Inches by it self, that is, by 4 Feet 5 Inches,

ches, it produces 19 Feet, 6 Inches, 1 Part, for the Superficial Content of the Side; which Product multiplied by the ſaid 4 Feet, 5 Inches, produces 86 Feet, 1 Inch, 10 Parts, and 5 Seconds, for the Content required.

Example.

	F.	I.	P.	S.	T.
The Side given, is	4	5	0	0	0
Which multiply by it ſelf . . .	4	5	0	0	0
	16	2	1		
	1	8			
	1	8			
The Superficial Content of the Side, is	19	6	1		
Which multiply by the Depth or Side	4	5			
	76	2	6	5	
	2				
	7	11	4		
The Solid Content required, is .	86	1	10	5	

PROP.

Prop. III. *Fig. II.*

Suppose in Fig. II. A B C D *to be a streight piece of Timber, terminated at both ends by two equal long Squares, (which may be called the Bases) the length of which piece* B D *is* 3 *Feet,* 4 *Inches, and the broader side of the Base* A B 1 *Foot* 8 *Inches, and the narrower Side* B C 1 *Foot,* 1 *Inch, and the Content of this piece be required.*

FIrst find the Superficial Content of the Base or End, By multiplying the broader Side 1 Foot 8 Inches, by the narrower Side 1 Foot 1 Inch, the Product is 1 Foot, 9 Inches, 8 Parts, for the Superficial Content of the End; which 1 Foot, 9 Inches, 8 Parts, being multiplied by the Length 3 Feet, 4 Inches, Produces 6 Feet, 2 Parts, 8 Seconds, for the Solid Content of that piece of Timber.

Exam-

Example.

	F.	I.	P.	S.	T.
The broader Side of the Base A B, is	1	8	0	0	0
The narrower Side B C, is	1	1	0	0	0
Being multiplied.	1	8			
		1	8		
The Superficial Content of the Base, is	1	9	8		
Which multiply by the Length A E .	3	4			
	3	4			
	2	3			
		3	2	8	
		2			
The Solid Content required, is . .	6	0	2	8	

PROP. IV.

Suppose a piece of Timber or Stone, whose Length is 3 Feet, 4 Inches, be terminated at both ends by two equal Quadrats (or Squares) the Side whereof is 1 Foot, 7 Inches, what is the Content of such piece?

FIrst find the Content of one end, or Base, by multiplying 1 Foot 7 Inches, by 1 Foot 7 Inches, it produces 2 Feet, 6 Inches, and 1 Part, for the Content; then multiply the Content of the End by the Length 3 Feet 4 Inches, and it produces 8 Feet, 4 Inches, 3 Parts, and 4 Seconds, for the Solid Content.

An

An Advertiſement.

The uſual way among many Men, to meaſure Timber whoſe Sides are unequal, (although it be a falſe way) is this: They uſually add the Breadth and Depth together, and take the half for the Side of a mean Square, and after they have found the Content of that mean Square, as they call it, (falſe Square, as I term it) they multiply it into (or by) the Length, and the Product they conclude is the Content of the Piece.

Let us compare this falſe way with the true way.

Suppoſe the piece of Timber in the preceding *Prop. III.* whoſe Sides are unequal, to wit, one Side of the Baſe (or End) A B, is 1 Foot, 8 Inches, and the other Side of the ſame Baſe (or end) B C, is 1 Foot, 1 Inch.

Example of the falſe way.

Firſt, They add the two Sides together, namely, 1 Foot, 8 Inches, and 1 Foot, 1 Inch, which makes 2 Feet, 9 Inches, the half whereof is 1 Foot, 4 Inches, and 6 Parts, which they multiply by it ſelf, namely, 1 Foot, 4 Inches, and 6 Parts, and the Product is 1 Foot, 10 Inches, 8 Parts, and 3 Seconds, which they alſo multiply by the Length 3 Feet, 4 Inches, and the Product is 6 Feet, 3 Inches, 7 Parts, and 6 Seconds, for the Content, which is more than the true Content by above a quarter of a Foot, and the more unequal the two Sides are, the more falſe this way of Meaſuring gives the Content.

Of

Of Timber or Stone, consisting of 3, 5, or more Sides.

PROP. V. *Fig. III.*

Admit Fig. III. *to be a streight piece of Timber or Stone, whose Length* AB *is* 30 *Feet, to be terminated at both ends with two equal Equilateral Triangles, the Side whereof* GF *is* 12 *Feet; the Content whereof is required.*

Take this as a General Rule for Regular Polygons.

First, measure all the Sides round about, or girt them with a Line, then take half that length for one sum, or breadth; then find the Center H of the End or Base, which you may do by finding the middle of the Base Line, as also of one Side; and from the Angles opposite to the Base Line, and that Side, draw two Lines to the middle of them, and where those Lines intersect, there is the Center of the Triangle, as in the Figure at H; then take the nearest distance from the Center to one of the Sides for the other Sum or Depth; then multiply that Breadth and Depth one by the other, and the Product is the Superficial Content of the End or Base of the Piece, which Content multiplied by the Length of the Piece, produces the Solid Content.

Exam-

Example.

The Girt of the 3 Sides is 36 Feet, the half whereof is 18 Feet, which multiply by the neareſt ſpace from the Center to one Side, which is 3 Feet, 5 Inches, 6 Parts, 10 Seconds, and the Product is 62 Feet, 4 Inches, and 3 Parts, for the Content of the Baſe, or end of the Piece; which multiplied by the length of the Piece 30 Feet, produces 1870 Feet, 7 Inches, and 6 Parts, for the Content of the Piece.

The Operation.

	F.	*I.*	*P.*	*S.*	*T.*
The 3 Sides being added is 36 F. the half whereof, is	18	0	0	0	0
Which multiply by the neareſt ſpace from the Center, to wit, .	3	5	6	10	0
	54	9	15		
	7	6			
The Superficial Content of the Baſe, is	62	4	3		
Which multiply by the Length . .	30				
	1860	7	6		
	10				
The Solid Content of the whole Piece, is	1870	7	6		

Other-

Otherwise thus:

Find the Perpendicular of the Triangle, by the 4*th* Prop. of measuring of Plains, then multiply the Perpendicular by half the Base, the Product whereof being multiplied into the Length, gives the Solid Content.

Example.

	F.	I.	P.	S.
The Perpendicular is	10	4	8	6
Which multiply by half the Base . .	6			
	60	4	3	
	2			
The Content of the End, is . . .	62	4	3	
Which multiply by the Length . .	30			
	1860	7	6	
	10			
The Content (as before) is . .	1870	7	6	

PROP.

Prop. VI. *Fig. IV.*

Admit Fig. IV. *were a ſtreight piece of Timber, terminated at both ends with two equal Pentagons, whoſe Side* AB *is* 12 *Inches, and the neareſt diſtance from the Center* C *to the middle of the Side at* D, *is* 8 *Inches, and the Length* EF 15 *Feet, and the Content required.*

Example.

	F.	I.	P.
The 5 Sides added together is 5 F. the half whereof is	2	6	0
Which multiply by CD	0	8	
	1	4	
		4	
The Content of the Baſe, is	1	8	
Which multiply by the Length	15		
	15		
	10		
The Solid Content is	25		

Of

Of Squared Timber or Stone, being bigger at one end than at the other, in which form ſome Timber Trees grow; and being fell'd, are ſo hewn and brought to a Square.

PROP. VII. *Fig. V.*

Admit Fig. V. *to be a ſtreight piece of Timber or Stone, terminated at both ends with two unequal Squares; the Side of the greater Square (or end)* AB *is* 1 *Foot,* 8 *Inches, and the Side of the leſſer Square (or upper end)* EF, *is* 1 *Foot,* 3 *Inches, and the length* AE, 20 *Feet; what is the Content?*

The Rule is this:

1. FInd the Content of the lower end (or greater Square) ABC. 2ly. Find the Content of the upper end (or leſſer Square) EGDF. 3ly. Multiply one Content by the other. 4ly. From that Product extract the Square Root. 5ly. Add this Square Root and the whole Content of both the Baſes into one Sum. 6ly. Multiply this Sum by one Third part of the Altitude (or length) of the Piece, and the Product will be the Solid Content required.

Which Rule is demonstrated by Mr. *Oughtred* for measuring a Segment of a Pyramid, in *Ch.* 19. *Prob.* 21. of his *Clavis Mathematica*.

Example.

First, Find the Superficial Content of the lower end A B C, by multiplying 1 Foot, 8 Inches, by 1 Foot, 8 Inches, it produces 2 Feet, 9 Inches, 4 Parts, for the Superficial Content of the lower end.

Secondly, Find the Content of the upper end (or lesser Square) E G D F, by multiplying 1 Foot, 3 Inches, by 1 Foot, 3 Inches, the Product whereof is 1 Foot, 6 Inches, and 9 Parts, for the Superficial Content of the lesser Square (or upper end.)

Thirdly, Multiply one Content by the other, to wit, 1 Foot, 6 Inches, and 9 Parts, by 2 Feet, 9 Inches, and 4 Parts, and the Product is 4 Feet, 4 Inches, and 1 Part.

Fourthly, Extract the Square Root of 4 Feet, 4 Inches, and 1 Part, which Root is 2 Feet, and 1 Inch.

Fifthly, Add this Square Root and the Content of both the Bases together, all three in one Sum, and the Product is 6 Feet, 5 Inches, and 1 Part.

Sixthly, Multiply the last Sum by a third part of the length, namely, 6 Feet, 8 Inches, and the Product will be 42 Feet, 9 Inches, 10 Parts, and 8 Seconds, for the Solid Content required.

The Operation.

	F.	I.	P.	S.
The Side AB is	1	8	9	0
Which ſquare, or multiply by it ſelf .	1	8	9	0
	1	8		
		8		
		5	4	
The Superficial Cont. of the lower end, is	2	9	4	
The Side of the leſſer Square, or upper end, is	1	3		
Which ſquare, or multiply by it ſelf .	1	3		
	1	3	9	
		3		
The Superficial Cont. of the upper end, is	1	6	9	
Which mult. by the Cont. of the lower end	2	9	4	
Then Product of one end multiplied by the other, is	4	4	1	
The Square Root whereof, is . . .	2	1		
To which, add the Cont. of the lower end	2	9	4	
Alſo add the Content of the upper end	1	6	9	
And the Product is	6	5	1	
Which mult. by one Third of the length	6	8		
	36	3	4	
	2	6	6	
	4			8
The Solid Content required, is . .	42	9	10	8

An Advertisement.

The usual way which a great many Men use in measuring such a piece of Timber or Stone, is this, which is a false way, and finds a Content less than the true Content.

They take the Sides of the Square about the middle of the Piece, and this Square of the middle of the Piece they suppose to be a mean Square between the greater end and the lesser end; then they multiply the Content of this Square, by the Length of the Piece.

Let us compare this false way with the true way, and see what difference there will be between the Content of the Piece found by the foregoing 7*th* Prop. and the Content of the same Piece found according to the usual (but false) way.

Example.

The side of the Square taken in the middle of the Piece is 1 Foot, 5 Inches, and 6 Parts, which multiplied by it self, produces 2 Feet, 1 Inch, 6 Parts, and 3 Seconds, for the Content of the Mean Square, as they account it, which Content they multiply by the length of the Piece, 20 Feet, and it produces 42 Feet, 6 Inches, and 5 Parts, for the Solid Content, which is above a quarter of a Solid Foot less than the true Content found before, and the more the Tree (or Solid whatsoever) diminisheth, the more the Content measured this false way will want of the true Content.

Exam-

Example.

Suppose a piece of Timber, being the same length 20 Feet, were 1 Foot, 8 Inches square, at the lower (or greater) end; and 1 Foot square at the upper (or lesser) end, and the Content required.

First, We will find it the true way.

	F.	I.	P.	S.
The Content of the greater end (whose Side is 1 F. 8 I.) is	2	9	4	0
The Content of the lesser end (whose Side is 1 F.) is	1	0	0	0
Which being multiplied, produces . .	2	9	4	
The Square Root whereof, is	1	8		
To which, add the Content of the greater end	2	9	4	
Also add the Content of the lesser end	1	0	0	
The Product is	5	5	4	
Which multiply by $\frac{1}{3}$ part of the length	6	8		
	30	3	4	
	2	6	2	8
	3	4		
		2		
The true Solid Content, is	36	3	6	8

Now let us see what the usual (but false) way will produce.

	F.	I.	P.
The side of the Square taken in the middle, is	1	4	0
Which multiplied by it self, namely, . . .	1	4	0
	1	4	
		4	
		1	4
The Content of the mean Square, is . .	1	9	4
Which multiply by the length	20		
	20		
	15	6	8
The Content (the false way) is	35	6	8

Which Content is less than the true Content by almost $\frac{1}{4}$ of a Solid Foot, which in measuring of a great quantity of Paper-grown Timber, would be considerable.

Of PYRAMIDS.

A *Pyramid* is a Solid, comprehended under plain Surfaces, and from a Triangular, Quadrangular, or any Multangular Base, diminisheth equally less and less, 'till it finish or end in a Point at top; such are the Spires of some Steeples.

Note: If a Pyramid be cut into two Segments or Parts by a plain Parallel to the Base, one of those Segments or Parts will be a Pyramid (to wit, that which diminisheth to a Point) and the other Segment or Part will have two unequal Bases, such as is *Fig. V.* the manner of finding its Content is shewn at *Prop VII.*

PROP.

PROP. VIII. *Fig. VI.*

Admit Fig. VI. *were a Pyramid, whose Base is an Equilateral Triangle, and one side of the Base* AB *is* 1 *Foot, and the Perpendicular of the Base* CD 10 *Inches,* 4 *Parts,* 6 *Seconds, and the length of the Pyramid* RS 8 *Feet, what is the Content of this Pyramid?*

The Rule for finding the Content, is this.

MUltiply the Content of the Base, by one Third part of the height of the Pyramid, and the Product is the solid Content. (By Height, is to be understood a Perpendicular Line, that falleth from the top of the Pyramid to the middle or Center of the Base: You are not to understand by Height, the length of the slope Line on the outside RS, for that is longer than the Altitude or Height.) And before we proceed any farther, it will be convenient to shew how to find the height of any Pyramid.

How to find the Height of any Pyramid.

First, Square the Semidiameter of the Base.

Secondly, Square the length of the Pyramid, which is the slope Line RS.

Thirdly, Subtract the lesser Square out of the greater.

Fourthly, Find the Square Root of the remainer, and that is the Height required.

Example.

First, The Semidiameter of the Base EF, is 3 Inches, 5 Parts, 7 Seconds, which being squared, is 12 Inches, (or 1 of those Inches whereof 12 make a Superficial Foot.)

Secondly, The length of the Pyramid R S, is 8 Feet, which being squared, is 64 Feet.

Thirdly, The lesser Square 1 Inch, being taken out of the greater Square 64 Feet, there remains 63 Feet, and 11 Inches.

Fourthly, The Root of 63 Feet, and 11 Inches, being extracted, is 7 Feet, 11 Inches, 11 Parts; which is the height of the Pyramid required.

And this Rule is general, for the finding the height of any Pyramid, whether Triangular, Quadrangular, or Multangular, always remembring to find the Center of the Base, that so you may have the Semidiameter thereof; which Centers are found, by drawing Lines from the Angles, to the Middle of the Sides opposite to them.

The Operation.

	I.	P.	S.	T.
The Semidiameter of the Base, being EF, is	3	5	7	0
You must square, that is, multiply it by	3	5	7	
	9	2	1	
	1	3	2	1,1
	1	3	2	1,1
		1	9	
		1	9	
The Square of the Semidiameter, is	1	0	0	0	10

The

	F.	I.	P.	S.
The length of the Pyramid R S, is	8	0	0	0
Which ſquare, or multiply by it ſelf	8			
And the length being ſquared, is	64	0	0	0
Out of which, ſubtract the Square of the Semidiameter		1		
And there remains	63	11	0	0
The Square Root whereof (being the height) is	7	11	11	11 +

Thus having found the height of the Pyramid, we will proceed in the next place, to find the Content of the Baſe.

	F.	I.	P.	S.
The Perpendicular C D, is	0	10	4	6
Which multiply by half the Baſe A C		6		
The Content of the Baſe, is	0	5	2	3
Which multiply by $\frac{1}{3}$ of the height	2	7	11	11 +
	10	4	1	10
	2	11	6	2
		1	2	2
		4	7	
			1	9
			4	7
The Solid Content of the Pyramid, is	1	1	9	11	6

PROP.

Prop. IX. *Fig. VII.*

Admit Fig. VII. *were a Pyramid, whose length* AE *is* 15 *Feet, and the Base of it* ABCD *a Square, whose Side* AB, *or* CD, *is* 2 *Feet*, 6 *Inches, what is the Content?*

First find the Altitude, or Height, which is the prick Line EF, by the Rule in the foregoing *Prop. VIII.* then multiply one Third part of the Height by the Content of the Base ABCD, and the Product is the Content required.

The Operation.

	F.	I.	P.	S.	T.
The Side of the Square of the Base AB, or CD, is	2	6	0	0	0
Which multiply by it self . . .	2	6			
	4	3			
	1				
	1				
The Content of the Base, is . .	6	3	0	0	0
Which multiply by $\frac{1}{3}$ of the Height	4	11	9	6	4
	24	2	9	3	1
	1	6	2	1	6
	5	4	6	2	1
			3		
The Solid Content of the Pyramid, is	31	1	8	6	7

How

How to find the Third part of any Number

Suppoſe you were to find the one Third part of the eight of the Pyramid aforeſaid.

	F.	I.	P.	S.	T.
Set down your Number, thus .	14	11	4	7	0
Then ſay, The $\frac{1}{3}$ part of 14 Feet, is .	4	8			
The $\frac{1}{3}$ part of 11 Inches, is		3	8		
The $\frac{1}{3}$ part of 4 Parts, is			1	4	
The $\frac{1}{5}$ part of 7 Seconds, is				2	4
Being added, the Product is . . .	4	11	9	6	4

Which is the Third part required.

After the ſame manner you may find the third Part of any other number; you may likewiſe find the Fourth Part, or fifth Part of any Number, after the ſame manner.

PROP. X. *Fig. VIII.*

Admit Fig. VIII. *were a Pyramid, whoſe length* A H *is* 15 *Feet, and the Baſe thereof a Pentagon, whoſe Side* A B *is* 2 *Feet,* 6 *Inches, and the Content required.*

First find the Content of the Baſe A B C D E (by the Rule delivered at the 9*th* Prop. of *Lib.* 5. of *Meaſuring of Plains*) which I will here repeat.

The

The Rule is this:

As 182, *Is to* 125: *So* is the Side of the Pentagon given, *To* the Semidiameter of a Circle inscribed within that Pentagon.

Then say, If 182 give 125, what will (the Side of the Pentagon) 2 Feet, 6 Inches, give)

	F.	I.
Multiply the second number . . .	125	6
By the third number	2	6
	250	
	62	6
And the Product is	312	6

Which divide by the first number 182, and the Quotient is 1 Foot, and 130 remains, which multiply by 12, and the Product is 1560, which divide by 182, and the Quotient is 8 Inches, and 104 remains, which multiply by 12, and the Product is 1248, which divide by 182, and the Quotient is 6 Parts, and 156 remaining, which multiply by 12, and the Product is 1872, which divide by 182, and the Quotient is 10 Seconds, and 52 remains, which multiply by 12, and the Product is 624, which divide by 182, and the Quotient is 3 Thirds; as you may see in the Margin: There is yet a

(130 (1 Foot.
182
130
12
260
130
1560
(104 (8 Inches
1560
182
Remainer

Remainer, but the value of it is not $\frac{1}{1000}$ part of an Inch; and in many Cases you will not need to proceed any farther than to Parts of Inches.

In the next place, there is the 312 Inches that belong to the 312 Feet, to be divided by 182; but because the Divisor is greater than the Dividend, multiply the Inches by 144, and the Product is 864 Seconds; then divide the 864 by 182, and the Quotient is 4 Seconds, which you must add to the 10 Seconds, and they make 1 Part, and 2 Seconds.

So is there found the length of the Semidiameter of the Circle described within the Pentagon, (whose Side was given) namely, 1 Foot, 8 Inches, 7 Parts, 2 Seconds, and 3 Thirds, which is the neareſt diſtance from the Center of the Pentagon, to either of the Sides.

104
12
208
104
1248

(156
x248 (6 Parts.
182
156
12
312
156
1872

Ø(5
x872 (10 Sec.
x822
x8

The next work will be, to find the Content of the Base of the Pyramid.

First,

	F.	I.	P.	S.
First, add the 5 Sides together, and they make 12 Feet, and 6 Inches, the half whereof, is . . .	6	3	0	0
Which multiply by the Semidiameter found, which is	1	8	7	2
	6	3	1	9
	4	2	6	1
		3	1	
And the Content of the Base ABCDE, is	10	8	8	11
Which multiply by a third Part of the height of the Pyramid . .	5			
	50		4	7
	3	4		
		3	4	
And the Solid Content of the Pyramid, is	53	7	8	7

I shall not instance any more Examples of Pyrami for if the Base consist of 6 or more Sides, the Cont of the Base may be found by the Rules delivere *Lib.* 5. and then by multiplying the Content of Base into one third part of the height of the Pyra the Product is the Solid Content.

Of Round Solids, such as are Cylinders, Cones, Spheres, Columns, &c.

PROP. XI. *Fig. IX.*

Admit Fig. IX. *were a Cylinder, (by Cylinder is meant, such a like Solid as is used in Gardens for the Rolling of Walks) whose length* A B *is* 6 *Feet, and the Diameter* B C, 1 *Foot and* 4 *Inches, and the Content required.*

The Rule for measuring such a Solid, is this:

MUltiply the Content of the Base B C, or A D, by the length A B, and the Product is the Solid Content.

First, let us find the Content of the Base, the Rule is this: *As* 452, (that is, four times 113) *Is to* 355: *So* is the Square of the Diameter, *To* the Content required.

	F.	I.	P.	S.
The Diameter is 1 F. 4 I. the Square of it, is	1	9	4	0
Which multiply by	355			
	355	118	4	
	266	3		
And the Product is	631	1	4	

Which

(179
452 (1 Foot.
452

179
12
358
179
2148

(340
2148 (4 Inches.
452

340
12
680
340
4080

(12
4080 (9 Parts.
452

Which divide by the first number 452, and the Quotient is 1 Foot, as you see in the Margin, and 179 remains; then continuing multiplying the Remainers by 12, and dividing the Products by 452, 'till you come to the Denomination of Thirds, as you see in the Margin; at last your Quotient is 9 Parts, and 12 remaining, which 12 is inconsiderate in value; but for the 1 Inch, and 4 Parts, that belong to the 631 Feet, and for the 12 that remains above your last Division, you may add 6 Thirds; for if you should continue multiplying and dividing the Remains, you would produce 6 Thirds.

	F.	I.	P.	S.	T.
Then the Content of the Base is found to be	1	4	9	0	6
Which must be multiplied by the length A B	6				
	6	4	6	3	
	2				
And the Product is	8	4	6	3	

Which is the Solid Content of the Cylinder.

Of a Cone.

A *Cone* is a Solid, which hath a Circle for its Baſe, from whence it grows equally leſs and leſs, (like a Sugar Loaf) 'till it finiſh or end in a point at top.

PROP. XII. *Fig. X.*

Admit Fig. X. *were a Cone, whoſe length* AC *is* 15 *Feet, and the Diameter of the Baſe* AB, 1 *Foot,* 4 *Inches, what is the Content thereof?*

THE Rule for finding the Solid Content of a Cone, is the ſame as for the finding the Solid Content of a Pyramid, to wit, by multiplying the Content of the Baſe by one third part of the Height.

Firſt, find the height of the Cone, (according to the Rule preſcribed in *Prop.* 8. of this Book) for the height of a Cone is found by the ſame Rule, that the height of a Pyramid is found.

	F.	I.	P.
1. The Semidiameter of the Baſe AB, is	0	8	0
Which ſquare (or multiply by it ſelf) . .	0	8	0
The Square of the Semidiameter, is . .	0	5	4

	F.	I.	P.	S.	T.
2ly. The length of the Cone AC, is	15				
Which square (or multiply by it self)	15				
	75				
	15				
The Product is	225				
3ly. Out of the Square of the length AC, subtract the Square of half the Diameter AB, to wit,		5	4		
And the Remainer is	224	6	8		
The Square Root whereof, is	14	11	9	11	*fere*
Which is the height of the Cone; of which height, we must take one third part, which is	4	11	11	5	8

Having found the Height, and taken a third part thereof, the next work will be, to find the Content of the Base.

Then say by the preceding Rule in *Prop. XI.* *As* 452, *Is* to 355: *So* is the Square of the Diameter, *To* the Content of the Base.

	F.	I.	P.
The Square of the Diameter, is	1	9	4
Which multiply by	355		
	355	118	4
	266	3	
And the Product is	631	1	4

Which

	F.	I.	P.	S.	T.
Which divide by 452, and you will find the Content of the Baſe to be (as in *Prop. XI.*) . . .	1	4	9	0	6
Which multiply by one third part of the height of the Cone . . .	4	11	11	3	8
	4	11	11	3	5
	1	4	8	3	
		3	8	8	3
		3	3	8	2
				1	8
				2	2
Thus the Solid Content of the Cone, is	6	11	8	2	8

Note: If a Cone be cut into two Segments (or Parts) by a plain Parallel to the Baſe; one of thoſe Parts will be a Cone, and the other Part will have two unequal Circles for the Baſes: As in *Fig. XI.* the upper part is a Cone, and the lower p rt is a Segment of a Cone, which hath two unequal Circles for the Baſes.

PROP. XIII. *Fig. XI.*

A Segment of a Cone may well be repreſented by a Tree that grows taper or diminiſhing.

ADmit the Segment in *Fig. XI.* were a round taper piece of Timber, whoſe length A B is 15 Feet, and the Diameter of the lower Baſe A C, 1 Foot, 4 Inches; and the Diameter of the upper end, or Baſe D B, is 1 Foot; what is the Content of this Piece?

The length A B being 15 Feet, and the Diameter A C being 1 Foot, 4 Inches; the Length, and likewiſe the Diameter, is the ſame, as was the Cone in the laſt Propoſition.

For the finding the Content of this piece of Timber (or Segment of a Cone) you muſt have reſpect to the Rule delivered in the preceding *7th* Prop. of this Book, for the ſame Rule that ſerveth for ſquare Timber, whoſe Baſes or Ends are unequal, ſerveth likewiſe for round Timber or Bodies, whoſe Baſes or Circles at the ends are unequal.

Example.

The Content of the lower Baſe A C, is by the laſt foregoing Propoſition found to be 1 F. 4 I. 9 P. 0 S. 6 T.

Now we muſt find the Content of the upper Baſe, or leſſer end B D, whoſe Diameter is 1 Foot.

And becauſe the Square of the Diameter is but 1 Foot, the Content of the Baſe cannot be ſo much as 1 Foot; therefore we muſt ſtate the Queſtion in Inches, ſaying thus:

If 452 give 355, what will the Diameter, whoſe Square is 12 Inches, give?

For if you remember, the Rule is, *As* 452, *Is* to 355: *So* is the Square of the Diameter, *To* the Content of the Circle of the Baſe (or end of the Piece.)

There-

	Inches.
Therefore multiply the second number . .	355
By the third number in the Question, to wit, .	12
	710
	355
And the Product is	4260

Which divide by the first number 452, and the Quotient is 9 Inches, and $\frac{192}{452}$ of an Inch; which, when reduced as is shewn before, by multiplying the Remainers by 12, and dividing the Products by 452, you will find 9 Inches, 5 Parts, 1 Second, and 2 Thirds, for the Content of the upper end or Base of the Piece.

(192
4260 (9 Inches.
452

Having now the Content of both the Bases, the next work will be, to multiply them one by the other.

	F.	I.	P.	S.	T.	Fo.
The Content of the lesser Base BD, is	0	9	5	1	2	0
Which multiply by the Content of the greater Base AC .	1	4	9	0	6	0
	9	6	9	2	9	
	3	1	8	4		
		5	3	9	8	
			1	4	6	
					1	
					2	
The Product of the two Bases, multiplied, is	1	1	1	10	9	2

The Square Root whereof, is 3 I. 7 P. 6 S.

Which Square Root, and the Content of both the Baſes, muſt be added together.

	F.	I.	P.	S.	T.
The Square Root, is	0	3	7	6	0
The Content of the greater Baſe, is	1	4	9	0	6
The Content of the leſſer Baſe, is	0	9	5	1	2
And the Product of the Addition, is	2	5	9	7	8
	5				
Which ſhould be multiplied by $\frac{1}{3}$ part of the height, namely, 4 F. 11 I. 11 P. 3 S. 8 T. but becauſe $\frac{1}{3}$ part of the height, and $\frac{1}{3}$ part of the length, differ but by 8 Seconds, you may multiply it by $\frac{1}{3}$ part of the length, namely, 5 Feet, and the Product is	10				
	2	1			
		3	9		
			2	11	
				3	4
	12	5	0	2	4

Which is the Solid Content of the Segment of the Cone.

Of a Globe or Sphere.

A Globe or Sphere is a perfect round Body contained under one Circular Plain, the middle Point whereof is called the Center, from whence all ſtreight Lines drawn to the outſide, are of equal length, and are called Semidiameters, or rather Semiaxes.

PROP.

PROP. XIV. *Fig.* XII.

Let Fig. XII. *be a Sphere, whose Axis* AB *is* 8 *Feet, what is the Solid Content thereof?*

The Rule is this:

AS 21, *Is to* 11 (or, *As* 42, *To* 22): *So* is the Cube of the Diameter or Axis, *To* the Solid Content required.

	F.
The Axis is 8, and the Square of it	64
Which multiplied by the Axis	8
The Cube of the Axis, is	512

Then say by the Rule of Three: If 21 give 11, what will (the Cube of the Diameter) 512 give?

Multiply 512 by 11, and the Product is 5632, (as you see in the Margin) which divide by the first number in the Question, to wit, 21, and the Quotient is 268 Feet, and $\frac{4}{21}$ of a Foot; which being reduced, is 2 Inches, 3 Parts, and 5 Seconds.

So that the Solid Content of the Sphere, whose Axis is 8 Feet, is 268 Feet, 2 Inches, 3 Parts, and 5 Seconds.

512
11
512
512
5632

xx
x47(4
5632 (268 Feet.
xxxx
xx

PROP. XV.

A Sphere is given, whose Solid Content is 268 Feet, 2 Inches, 3 Parts, and 5 Seconds, what is the length of the Axis?

The Rule is this,

AS 22, *Is* to 42: *So* is the Solid Content given, to wit, 268 Feet, 2 Inches, 3 Parts, 5 Seconds; *To* the Cube of the Axis, whose length is required.

State the Question, thus: If 22 give 42, what will 268 F. 2 I. 3 P. 5 S. give?

F.	I.	P.	S.
268	2	3	5
42			
536			
10727	10	6	
		17	6
11263	11	11	6

Multiply the third number by the second, (as in the Margin) and the Product is 11264 Feet *ferè*; which divide by the first number in the Question 22, and the Quotient is 512, (which is the Cube of the Axis) the Cube Root whereof, is 8 Feet, which is the length of the Axis required.

The briefest way to Extract Square and Cube Roots, is by a Table of Logarithms.

x24
x1264 (512
2222
22

PROP.

PROP. XVI. *Fig.* XII.

Admit in Fig. XII. CED *were a Segment (or Portion) of a Sphere, whose Segment of the Axis* EF *is* 2 *Feet, (the whole Axis* AB *being* 8 *Feet) and the Chord (or Subtense) of the Segment* CD, *is* 6 *Feet,* 11 *Inches, and* 2 *Parts, what is the Solid Content of this Portion of the Sphere?*

The Rule is this:

FIrst, encrease the Altitude of the other Segment (not given) by half the Axis.

Then say by the Rule of three: *As* the Altitude of the other Segment not given, *Is* to the Altitude (or Height) of the given Segment (or Portion): *So* is the Altitude of the other Segment encreased by half the Axis, *To* a fourth Proportional.

Secondly, Square half the Chord (or Subtense) of the given Segment, and multiply the Square of that half Chord by the fourth Proportional found by the first Work (or Rule).

Then say by the Rule of Three: *As* 21, *Is* to 22: *So* is the Product of the Square of half the Chord of the Segment given, multiplied by the fourth Proportional (found as above), *To* the Solid Content of the Segment given.

Example.

Example.

The Segment not given is CDG, whose Altitude FG is 6 Feet, which must be encreased by half the Axis, namely, 4 Feet, and then it is 10 Feet.

Then say: *As* (the Altitude of the other Segment not given, which is) 6 Feet, *Is* to (the Altitude of the given Segment) 2 Feet: *So* is (the Altitude of the other Segment not given, being encreased by half the Axis, to wit, 6 Feet more 4 Feet, that is) 10 Feet, *To* a fourth Proportional required.

Let us find this fourth Proportional.

First, Multiply 10 F. by 2 F. and the Product is 20 F. which divide by 6 F. and the Quotient is 3 F. $\frac{2}{6}$ parts of a Foot, that is, 3 F. 4 I. which is the fourth Proportional.

In the next place, Square half the given Chord CF; the whole Chord CD, is 6 F. 11 I. 2 P. the half whereof is 3 F. 5 I. 7 P. *ferè*, the Square whereof is 12 F. 0 I. 1 P. 2 S. + which must be multiplied by the fourth Proportional found, namely, 3 F. 4 I. and the Product is 40 F. 0 I. 3 P. 10 S. 8 T.

Then say by the Rule of Three: *As* 21, *Is* to 22: *So* is (the Square of half the given Chord, multiplied by the fourth Proportional, found as before) 40 F. 0 I. 3 P. 10 S. 8 T. *To* the Content required.

Then multiply the third number by the second, and the Product is 880 F. 7 I. 1 P. 6 S. 8 T. which divide by the first number 21, and the Quotient is 41 F. and $\frac{12}{21}$ of a Foot, which (when reduced) is

41

41 F. 11 I. *ferè*, which is the Solid Content of the Segment required.

For note, That a Sphere is equal to two Cones, having their Height and the Diameter of their Base the same with the Axis of the Sphere: Or, which is all one, a Sphere is two third Parts of a Cylinder, having the Height and the Diameter of the Base the same with the Axis of the Sphere.

But in case, you have only the Segment of a Sphere given, and not the Axis of the whole Sphere, then you must find the Axis.

PROP. XVII. *Fig. XII.*

The Segment or Portion of a Sphere being given, and the Axis required.

The Rule is this:

SQuare one half of the Chord of the Segment, and the Product divide by the Altitude of the given Segment; then to the Quotient add the Altitude of the given Segment, and the Product is the length of the Axis required.

Example.

Admit in Fig. XII. *the Segment given to be* CED, *and the Axis* AB *required to be found.*

The given Chord CD is 6 F. 11 I. 2 P. the half whereof is 3 F. 5 I. 7 P. which being squared, is 12 F. which divide by (the Altitude EF) 2 F. and the

the Quotient is 6 F. To which add the Altitude of the Segment (EF) 2 Feet, and the Product is 8 F. which is the length of the Axis of the whole Sphere, whereof CED is a Segment.

PROP. XVIII.

The Axis of a Sphere being given, to find the Superficial Content.

One Rule is this:

AS 7, *Is* to 22: *So* is the Square of the Diameter, *To* the Superficial Content required.

Example.

Let the Diameter or Axis given be 8 Feet, which being squared, is 64 Feet.

Then say by the Rule of Three: If 7 give 22, what will 64 give?

Multiply 64 by 22, and the Product is 1408; which divide by 7, and the Quotient is 201 Feet, and $\frac{1}{7}$ of a Foot; which reduced, is 201 F. 1 I. 8 P. 7 S. *ferè*, which is the Superficial Content of the Sphere.

On the contrary, If the Superficial Content of a Sphere be given, and the Axis required,

The Rule is this:

As 22, *To* 7: *So* is the Superficial Content given, *To* the Square of the Axis.

Example.

Example.

The Content given is 201 F. 1 I. 8 P. 7 S. what is the Axis?

Multiply the Content given, by 7, and the Product is 1408, which divide by 22, and the Quotient is 64, which is the Square of the Diameter; the Square Root whereof is 8, the Axis or Diameter required.

PROP. XIX. *Fig. XIII.*

A Portion or Segment of a Sphere being given, to find the Content of the Convex Superficies thereof.

ADmit in *Fig. XIII.* B A C to be a Portion of a Sphere, whose Altitude is A E, 4 Feet, and whose Base or Chord is B C, 8 Feet, and the Content of the Convex Superficies is required.

Draw the streight Line A B from the Pole A, to the Base in B; then, I say, the Content of a Circle, whose Radius (or Semidiameter) is A B, is equal to the Content of the Convex Superficies of the Portion B A C.

Let us try.

The Semidiameter A B, is 5 F. 7 I. 10 P. 6 S. Now the Content of a Circle, described from that Semidiameter, (being found by *Prop. XVII.* of *Measuring of Plains*) will be 100 F. 6 I. 6 P. which is the Content of the Convex Superficies of the Portion B A C, which was required.

Which

Which may be proved by the laſt foregoing Propoſition; for the Content Superficial of a Sphere, whoſe Diameter is 8 F. was found to be 201 F. 1 I. 8 P. +: Now this Portion B A C is half the Sphere of the ſame Axis, therefore double the Content of it, which was found to be 100 F. 6 I. 6 P. and the Product is 201 F. and 1 I. for the Content of the whole; which is the ſame that was found by the preceding Propoſition, within (or want) 8 P. which want of the 8 Parts is cauſed by the length of the Line A B, it being a ſmall matter more than 5 F. 7 I. 10 P. 6 S.

The truth of this laſt Propoſition, that is to ſay, That the Circle deſcribed by the Radius A D, or A B, is equal to the Convex Superficies of the Portion of a Sphere B A C, is demonſtrable from the compariſon of Motion, thus:

Let the Plain A E B D be underſtood to make a Revolution about the Axis A E; and it is manifeſt, that by the ſtreight Line A D, a Circle may be deſcribed; and by the Arch A B, the Superficies of a Portion of a Sphere; and laſtly, by the Subtenſe A B, the Superficies of a right Cone will be deſcribed. Now ſeeing both the ſtreight Line A B, and the Arch A B, make one and the ſame Revolution, and both of them have the ſame extream Points A and B; the cauſe why the Spherical Superficies which is made by the Arch, is greater than the Conical Superficies which is made by the Subtenſe, is, That A B the Arch, is greater than A B the Subtenſe; and the cauſe why it is greater, conſiſts in this: That although they be both drawn from A to B, yet the Subtenſe is drawn ſtreight, but the Arch angularly, namely, according to that Angle which the Arch makes with the Subtenſe, which Angle is equal to the Angle D A B, (for an Angle of Contingence

tingence adds nothing to an Angle of a Segment.) Wherefore the Magnitude of the Angle DAB is the cauſe why the Superficies of the Portion deſcribed by the Arch A B, is greater than the Superficies of the right Cone deſcribed by the Subtenſe A B.

Again, The cauſe why the Circle deſcribed by the Tangent A D, is greater than the Superficies of the right Cone deſcribed by the Subtenſe A B, (notwithſtanding that the Tangent and Subtenſe are equal, and both moved round in the ſame time) is this, That AD ſtands at right Angles to the Axis, but AB obliquely; which obliquity conſiſts in the ſame Angle DAB. Seeing therefore the quantity of the Angle DAB is that which makes the exceſs both of the Superficies of the Portion and of the Circle made by the Radius A D, above the Superficies of the right Cone deſcribed by the Subtenſe A B; it follows, That both the Superficies of the Portion, and that of the Circle, do equally exceed the Superficies of the Cone. Wherefore, the Circle made by A D, or A B, and the Spherical Superficies made by the Arch A B, are equal to one another, which was to be proved.

Of the Proportions between a Cube, a Priſma, and a Pyramid; a Cylinder, Sphere, and Cone, whoſe Altitudes and Baſes are as follow.

	F.	I.	P.	S.	T.
A *Cube*, whoſe Side or Baſe is 1 F. 4 I. the Content is . . .	2	4	5	4	0

	F.	I.	P.	S.	T.	Fo.
A *Prisma*, having its Base and Altitude severally equal to the Base of the aforesaid Cube, (or of any other Rectangular Parallelepipedon) is the half of it, and the Content is . . .	1	2	2	8	0	0
A *Square Pyramid*, having its Height and Base severally equal to the Base of the aforesaid Cube, is $\frac{1}{3}$ part of it, and its Content is .	0	9	5	9	4	0
A *Triangular Pyramid*, whose Height is as aforesaid, and each Side of the Base as afore, (that is, 1 F. 4 I.) the Solid Content is . . .	0	4	1	3	1	4
A *Cylinder*, having the same Height and Diameter with the Cube aforesaid, the Solid Content of such a Cylinder is	1	10	4	2	1	0
A *Sphere*, whose Axis is as aforesaid, its Solid Content is	1	2	10	9	4	8
A *Cone*, whose Altitude and Diameter of its Base are severally equal to the Side of the Cube aforesaid, its Solid Content is . .	0	7	5	4	8	4

Hence it appears, that a *Cube* is double the *Prisma*, and three times as much as the *Square Pyramid* of equal Base and Altitude.

The

The *Triangular Pyramid* is ſomething more than $\frac{1}{7}$ of the Cube.

The *Cylinder*, whoſe Diameter and Height is ſeverally equal to the height of the Cube, is in proportion to it as 11 to 14; or the Cylinder contains 11 of thoſe Parts whereof the Cube contains 14.

The *Globe* or *Sphere*, whoſe Axis is equal to the height of a Cube, contains 11 ſuch Parts, whereof the Cube contains 21; or the Sphere is in proportion to the Cube, as 11 to 21.

The *Cone*, whoſe Diameter and Altitude are ſeverally equal to the Altitude and Diameter of the Cylinder, is in proportion to the Cylinder, as 1 to 3.

Whence it appears, that the Sphere is $\frac{2}{3}$ of the Cylinder, and the Cone $\frac{1}{2}$ of the Sphere.

Of GAUGING.

GAuging is comprehended in the meaſuring of *Solids*, and there is only this difference between Gauging (or meaſuring of Veſſels), and meaſuring of other Solids: The Content of the latter is given in Feet, Inches, Parts, *&c.* but the Content of the former is given in Gallons, Quarts, Pints, *&c.*

But before you can give the Content in Gallons, *&c.* you muſt firſt find the Solid Content in Cubick Inches, Parts, *&c.* and having firſt found the Content in Inches and Parts, you muſt afterwards reduce them into Gallons, Quarts, and Pints, *&c.*

There are ſeveral methods ſhewn for the finding the Content of theſe Irregular Solids; but the method ſhewn by Mr. *Oughtred*, is generally eſteemed for one of the beſt; and therefore I ſhall make uſe of his method.

Most Liquid Vessels, such as are Pipes, Hogsheads, Barrels, Kilderkins, Firkins, *&c.* are made in form of a Spheroides, having the two ends equally cut off; and accordingly may be measured thus:

Measure the two Diameters of the Vessel in Inches, the one at the Bung-hole, the other at the Head, and also the length within; and by the Diameters found, find out the Area's, or Contents of the Circles: Then add together two third Parts of the Content of the greater Circle, and one third Part of the Content of the lesser Circle. Lastly, multiply the Product of these two Sums added together, by the length of the Vessel; so shall you have the Content of the Vessel in Cubick Inches.

Of which, 231 make a Wine Gallon; and 272, and 3 Parts, make an Ale or Beer Gallon, according to Mr. *Oughtred*, who would have a Gallon to consist of a number of Cubick Inches, the Square Root whereof is Palms $5\frac{1}{2}$.

That 231 Cubick Inches make a Wine Gallon, is the opinion of most Men; but the quantity of the Ale, or Beer Gallon, is not as yet fully agreed on: Formerly the Ale Gallon hath been accounted to contain 288 Cubick Inches, and $\frac{1}{4}$, or 9 Parts; but since the Excise, it is computed to contain but 282 Cubick Inches.

PROP.

Prop. XX.

Admit a Vessel to have the Diameter at the Bung 32 Inches, and at the Head 18 Inches, and the Length 40 Inches, what is the Content of this Vessel in Gallons, and Parts of a Gallon?

THese Dimensions being given in Inches, and the Content required in Inches, you must have respect to the Note at *pag.* 288. *lib.* 5.

We must, first of all, find the Content of the two Circles belonging to the two Diameters given, by the second Rule delivered at the 17*th* Proposition of the fifth Book.

	Inches.
The Square of the Diameter 32, is . .	1024
Which, according to the Rule, multiply by .	355
	5120
	5120
	3072
And the Product is	363520

Which divide by 452, (according as the aforesaid Rule directs) and the Quotient is (as in the Margin) 804 Inches, and $\frac{112}{452}$ Parts of an Inch; which reduced, is 804 I. 2 P. 11 S. 8 T. for the

```
 (1
 ~~1~~9(12   I.
~~3~~63~~5~~2~~0~~ (804
 ~~4~~5~~2~~2~~2~~
  ~~4~~5~~5~~
   ~~4~~
```

Content of the greater Circle, to wit, the Circle at the Bung, whose Diameter was 32 Inches.

	Inches.
In the next place, the Square of the Diameter 18 Inches, is	324
Which multiply by	355
	1620
	1620
	972
And the Product is	115020

Which divide by 452, (as in the Margin) and the Quotient is 254 Inches, and $\frac{212}{452}$ of an Inch; which reduced, is 254 I. 5 P. 7 S. 6 T. and is the Content of the lesser Circle at the Head, whose Diameter was 18 Inches.

```
(2
2∅(1
246z(2
xx5∅z∅ (254 Inches.
45zzz
455
4
```

The next work is, to take the two third Parts of the Content of the greater Circle, and one third Part of the Content of the lesser Circle, and add them together.

The

	I.	P.	S.	T.	
The Content of the greater Circle, is	804	2	11	8	
The Content of the leſſer Circle, is	254	5	7	6	
Two third Parts of the greater Circle, is	536	1	11	9	+
And one third Part of the leſſer Circle, is	84	9	10	6	
Which being added, the Product is	620	11	10	3	
Which, multiply by the length	40				
	24800	33	4		
	36	8	10		
The Content of the Veſſel in Cubick Inches, is	24839	6	2		

The next Work will be, to find how many Gallons is contained in 24839 Cubick Inches.

And if you would know the Content in Wine Meaſure, divide the number of Inches by 231, and the Quotient gives you the Content in Wine Gallons.

But if you would know the Content in Ale Meaſure, then divide the number of Cubick Inches by 282, and the Quotient gives you the Content: Or if you would know the Content in Ale Meaſure, according as the Gallon was accounted (before the Exciſe), to contain 288 Cubick Inches, and $\frac{1}{4}$ (or 9 Parts). After you have reduced the Inches into Wine Meaſure, then multiply the Content in Gallons (and Parts, if there be any) by 4, and divide the Product by 5, and the Quotient gives you the Content re-

quired; for 231 being multiplied by 5, produces 1155 I. which being divided by 4, the Quotient is 288 I. and $\frac{1}{4}$, (or 9 P.) for 231 bears the same Proportion to 288 Inches and 9 Parts, that 4 does to 5.

Example.

Suppose you would know how many Gallons of Wine Measure is contained in 24839 Cubick Inches; divide the number given by 231, (as in the Margin) and the Quotient is 107 Gallons, and $\frac{22}{231}$ Parts of a Gallon; which reduced, is 107 Gallons and an half, and about $\frac{1}{4}$ of a Pint.

(1
x7(22
24839 (107 Gallons.
23xx2
233
2

Then if you would know how many Ale Gallons it contains, accounting 288 I. 9 P. to the Gallon; say,

As 5, *Is* to 4: *So* is the Content in Wine Gallons, *To* the Content in Ale Gallons.

Multiply 107 Gallons and an half by 4, and the Product is 430, which divide by 5, and the Quotient is 86 Gallons, and near a quarter of a Pint.

107 $\frac{1}{2}$
4
428
2
430

3
43∅ (86 Gal. $\frac{1}{4}$ Pint.
55

Or, if you would know how many Ale Gallons (of 288 I. 9 P. to the Gallon) there is in 24839 I. 6 P. without reducing it first to Wine Measure, do thus:

Multi-

Multiply the Content given in Inches, by 12, (to bring them into Parts) and the Product is 298068, to which, add the 6 Parts that are in the Content beſides the Inches, and the Product is 298074 P. (as in the Margin.)

I.	P.
24839	6
12	0
49678	
24839	
298068	
6	
298074	

Then reduce the Inches of the Gallon into Parts, by multiplying 288 by 12, and the Product is 3456; to which, add the 9 Parts, and the Product is 3465 Parts; laſtly, divide 298074 by 3465, and the Quotient is 86 Gallons, and $\frac{84}{3465}$ of a Gallon; which 84 Parts being reduced, is 7 Inches, which is not a quarter of a Pint, for a quarter of a Pint contains 9 Inches.

I.	P.
288	9
12	0
576	
288	
3456	
9	
3465	

(8
2887(4
298074 (86 Gallons.
34655
346

By the ſame method you may find the Content (of any number of Inches and Parts given) in Ale Meaſure, allowing (with Mr. *Oughtred*.) 272 Cubick Inches and 3 Parts, to the Gallon.

Now becauſe the finding the two third Parts of the Content of the Circle at the Bung, and the one third Part of the Content of the Circle at the Head, is ſomething tedious; I have therefore found out Proportional Numbers to ſhorten the Work, which are as follow.

As 1728 I. *Is* to 904 I. 9 P. 4 S. 5 T. 2 Fo. *So* is the Square of the Diameter at the Bung, *To* two third Parts of the Content of the Circle thereof.

Again: *As* 1728 I. *Is* to 452 I. 4 P. 8 S. 2 T. 7 F. *So* is the Square of the Diameter at the Head, *To* one third Part of the Content of the Circle thereof.

Having these Proportional Numbers, the Operation will be after this manner.

First, Square each Diameter.

Secondly, Multiply the Square of the Diameter at the Bung by 904 I. 9 P. 4 S. 5. T. 2 Fo.

Thirdly, Multiply the Square of the Diameter at the Head by 452 I. 4 P. 8 S. 2 T. 7 Fo.

Fourthly, Add these two Products together.

Fifthly, Divide the Product of the Addition by 1728.

Sixthly, Multiply the Quotient by the length of the Vessel, and the Product is the Content in Cubick Inches.

Example.

	I.	P.	S.	T.	Fo.
I. The Diameter at the Bung is 32 I. the Square whereof, is	1024	0	0	0	0
II. Which multiply by	904	9	4	5	2
	4096	341	4		
	92160		426	8	
	768			170	8
The Product is . . .	926495	6	0	10	8

III. The

	I.	P.	S.	T.	Fo.
III. The Diameter at the Head is 18 Inches, the Square whereof is . .	324	0	0	0	0
Which multiply by . .	452	4	8	2	7
	648	216	54		
	1620			189	
	129608				
	1				
The Product is . . .	146574	5	9	9	0
IV. To which, add the former Product . . .	926495	6	0	10	8
And the Sum is . . .	1073069	11	10	7	8

V. Divide this laſt Sum by 1728, and the Quotient is 620 I. and $\frac{1709}{1728}$ parts of an Inch, which being reduced, is 620 I. 11 P. 10 S. 5 T.

(17
362(0 I.
x∅73∅6(9 (620
x72888
x722
x7

VI. Multiply this laſt Sum by the length of the Veſſel 40 I. and the Product is (as in the Margin) 24839 I. 6 P. 8 S. 8 T. which is the Solid Content in Cubick Inches and Parts, and the ſame that was found the firſt way.

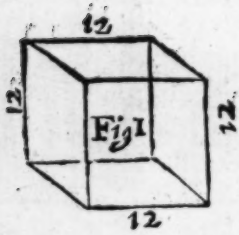
	I.	P.	S.	T.
	620	11	10	5
	40			
	24800	33	4	
	36	8	16	8
	24839	6	8	8

For

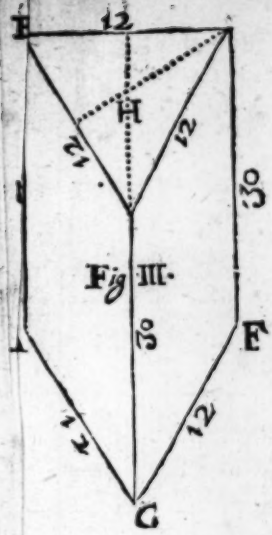
For Measuring or Gauging of *Brewers* Tuns or Vessels, whether they be square or round, or any other form, work thus;

First, find the Content in Cubick Inches, by the preceding Rules of measuring such Bodies; then divide that Content by 282 (the number of Inches agreed upon for a Gallon to contain since the Excise) and the Quotient is the Content in Gallons. Lastly, to bring the Gallons into Barrels, divide the number of them by 36, (the Gallons in one Barrel) and the Quotient gives you the Content of the Vessel or Tun, in Barrels.

FINIS.


12
12
12
12
Fig. I

A
B
C
D
E
1.8
1.1
1.8
1.1
3.4
3.4
1.8
1.1
Fig. II.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H
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F
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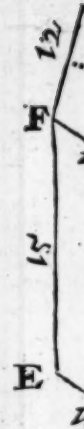
12
C
Fig. III.


12
F
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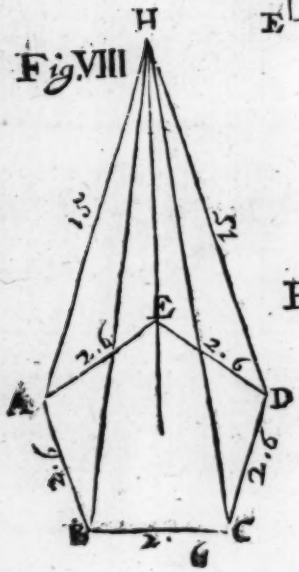
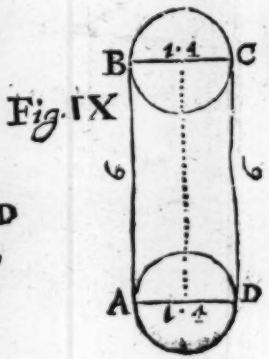
E


H
Fig. VIII
15
15
E
2.6
2.6
A
D
2.6
2.6
B
2.6
C


B
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C
Fig. IX
6
6
A
1.4
D

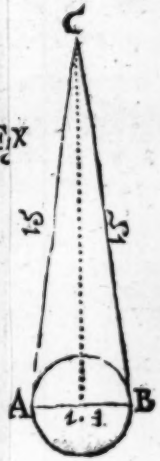
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Fig. X
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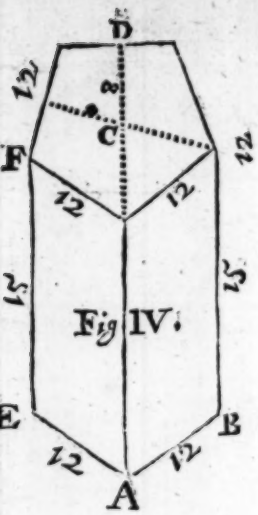
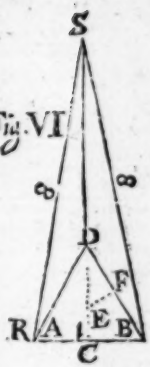
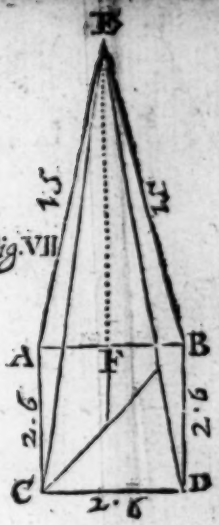
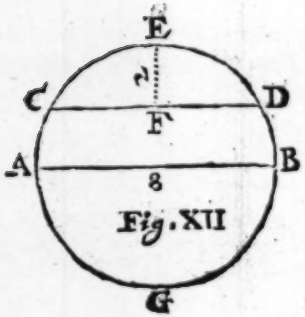
15
A
1.1
B

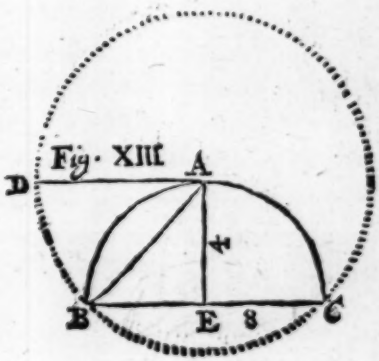
B
D
Fig. XI
A
1.4
C











Place this at the end of the Book.

ERRATA.

PAge 46. line 22. for *o — n*, read *o, n.* p. 79. l. 11. for *the*, read *a.* p. 91. l. 33. dele *ſtreight.* p. 92. l. 1. for 62, read 92. p. 173. l. 10. for *be*, read *to.* In the Title-page of the *Fourth Book*, line 6. for *Maſonſy*, read *Maſonry.* p. 194. l. 22. for 99, read 9. p. 223. l. 2. over 7, 4, 8, put *Fo. In. Pa.* Ibid. l. 11. the black Line under 22 . 2 . 0, ſhould be above it under 22 . 0 . 0. p. 234. l. 31. for 1 *Brickwork*, read 1 *Brick work.* p. 251. l. 15. for $\frac{1}{2}$, read 1 $\frac{1}{4}$. p. 263. in the Quotient of the Diviſion at the lower end, for 4 *l.* read 4 *d.* From p. 266. to p. 287. in the Titles read *Lib. V.* p. 269. l. ult. for 50 *F.* 9 *l.* 0 *P.* 11 *S.* read 50 *F.* 0 *l.* 0 *P.* 11 *S.*